

# $x^x$ ( $x$ の $x$ 乗) について

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## P.121

【例8】  $z = -\frac{3}{4}$  のとき、

$$\left(-\frac{3}{4}\right)^{\frac{3}{4}} = e^{\frac{3}{4}\log\left(-\frac{3}{4}\right)} = \left(\frac{3}{4}\right)^{\frac{3}{4}} \left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = \frac{2\sqrt{2}}{\sqrt[4]{27}} \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{2}{\sqrt[4]{27}}(-1+i)$$

$$\left(-\frac{3}{4}\right)^{\frac{3}{4}} = e^{\frac{3}{4}\log\left(-\frac{3}{4}\right)} = \left(\frac{3}{4}\right)^{\frac{3}{4}} \left\{\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right\} = \frac{2\sqrt{2}}{\sqrt[4]{27}} \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = \frac{2}{\sqrt[4]{27}}(-1-i)$$

$$\left(-\frac{3}{4}\right)^{\frac{3}{4}} = e^{\frac{3}{4}\log\left(-\frac{3}{4}\right)} = \left(\frac{3}{4}\right)^{\frac{3}{4}} \left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right) = \frac{2\sqrt{2}}{\sqrt[4]{27}} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{2}{\sqrt[4]{27}}(1+i)$$

$$\left(-\frac{3}{4}\right)^{\frac{3}{4}} = e^{\frac{3}{4}\log\left(-\frac{3}{4}\right)} = \left(\frac{3}{4}\right)^{\frac{3}{4}} \left\{\cos\left(-\frac{9\pi}{4}\right) + i\sin\left(-\frac{9\pi}{4}\right)\right\} = \frac{2\sqrt{2}}{\sqrt[4]{27}} \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = \frac{2}{\sqrt[4]{27}}(1-i)$$

## P.122

【例11】  $z = -\pi$  のとき、 $z^z$  の値のひとつは②より、

$$\begin{aligned} (-\pi)^{-\pi} &= \pi^{-\pi} \left\{ \cos(-\pi^2) + i\sin(-\pi^2) \right\} \\ &\doteq -0.0247567717 + 0.0118013091i \end{aligned}$$

## P.123

【例13】

よって、 $(2+3i)^{2+3i}$  の値のひとつは、

$$\begin{aligned} (2+3i)^{2+3i} &= \exp\left\{2\log\sqrt{13} - 3\tan^{-1}\frac{3}{2} + i\left(3\log\sqrt{13} + 2\tan^{-1}\frac{3}{2}\right)\right\} \\ &\doteq 0.6075666647 - 0.3087560181i \end{aligned}$$