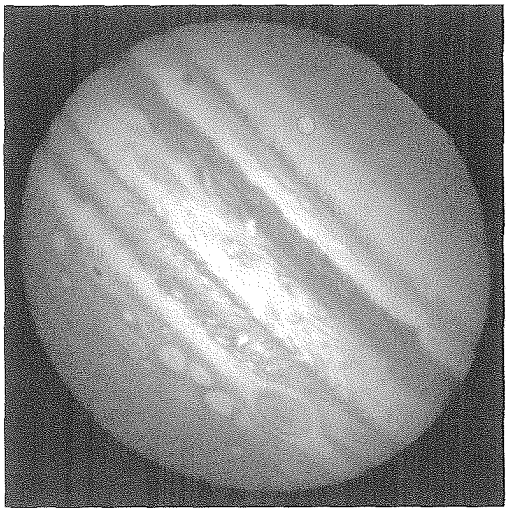
Measuring the Mass of Jupiter



Taken by the Hubble Space Telescope
February 13th 1995

Name:
Candidate Number:
Session: May 2004
Subject: Physics
Sponsor:
Words: 3931

Acknowledgements

Thanks to , my Extended Essay supervisor, for:

- Helping with the finding and selection of the topic
- *Giving access to a telescope and a digital camera and teaching me how* to take the photos
- Taking some of the photos when I was unable to
- Helping me with understanding the optics of a telescope
- Generally guiding me along the way

CONTENTS

Abstract

The aim of this investigation was to calculate the mass of Jupiter using Newton's derived version of Kepler's Third Law, $M_J = \frac{4\pi^2 r_S^{\ 3}}{G T_S^{\ 2}}$, and using observations of Io's orbit. Europa's orbit was used to verify the results but as the investigation did not focus upon Europa, the data is inadequate.

In order to calculate the mass, two pieces of data were needed, which was the radius and the period of the satellite's orbit. To do this, photographs were taken through a telescope of Jupiter from the 2nd April to the 11th April from 10pm to 2am. The photos taken were then analysed and the displacement of the satellite was plotted on a graph against time. Through trial and error, a sine curve was found that fit the points and through this the maximum displacement and the period of the satellite could be calculated.
However, as the maximum displacement was measured in pixels, this distance had to be converted into kilometers.

In order to do this, three items of data were needed, the conversion from pixels to an angular distance in the camera, the magnification of the telescope and the distance from Earth to Jupiter. By using optic diagrams and trigonometry, these three items were obtained and it was possible to convert the pixels into kilometers.

It was calculated that the mass of Jupiter is 2.65×10^{27}kg $\pm$ 8.2×10^{26}, using data from Io's orbit, and 3.93×10^{27}kg $\pm$ 9.1×10^{26}, using data from Europa's orbit, while reliable figures found show that the accepted mass of Jupiter is 1.9×10^{27} kg.[1]
However, as Europa was not the main focus of the investigation, the larger difference between the value found using Europa's orbit and the accepted value is acceptable.

WORDS: 290

---

[1] Students for the Exploration and Development of Space (SEDS).
The Nine Planets: A Multimedia Tour of the Solar System; Bill Arnett. Available from World Wide Web:<http://seds.lpl.arizona.edu/nineplanets/nineplanets/jupiter.html>

Introduction

In 1610, Galileo made a telescope and observed Jupiter and its four main satellites, Io, Europa, Ganymede and Callisto.
In 1619, Kepler used Brahe's observations to derive the Third Law of Planetary Motion. The Third Law states that "The ratio of the squares of the periods (the time needed for one revolution about the Sun) of any two planets revolving about the Sun is equal to the ratio of the cubes of their mean distances from the Sun."[2]

The Third Law derived by Newton from the Universal Theory of Gravitation and the laws of motion 50 years later, resulting in $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ where M is the mass orbited around, T and r, the period and radius of the orbit.

Newton's Derivation of the Third Law

$$F = m\frac{v^2}{r} = m\frac{4\pi^2 r}{T^2} \qquad F = \frac{GmM}{r^2}$$

$$m\frac{4\pi^2 r}{T^2} = \frac{GmM}{r^2}$$

$$\frac{4\pi^2 r^3}{T^2} = GM$$

$$\therefore M = \frac{4\pi^2 r^3}{GT^2}$$

Newton estimated the mass of Jupiter, applying his Universal Theory of Gravitation to observations of Callisto's orbit. His estimated mass of Jupiter was 300 times the mass of the Earth.[3] Newton's main limitation was his inaccurate data of Callisto's orbit and also of the Earth and the Moon. Newton estimated a value for GJ (Gravitational constant x Jupiter's mass) but as he did not have a value for G, he found J in terms of Earth masses.

Cavendish calculated the value of G using a torsion pendulum, which is $6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$.

---

[2] Giancoli, **Physics**, Fifth Edition, Prentice Hall International (UK) Limited, *London*, 1980 Pg 133
[3] Department of Earth Sciences, Simon Fraser University.
Global Geophysics; Dr. A. C. Calvert. Available from the World Wide Web:
<http://www.sfu.ca/earth-sciences/courses/317/Chap1/1-Introduction.htm>

This investigation is to calculate the mass of Jupiter like Newton, using his version of Kepler's Third Law of planetary motion rewritten as $M_J = \frac{4\pi^2 r_S^{\;3}}{G T_S^{\;2}}$.

To obtain data on the satellites of Jupiter's orbits, photos taken of Jupiter and its satellites were analysed to find the radius and the time period of the orbit. There was a time constraint that photos could only be taken in the two week school holiday as it was not possible to stay up late to take photos during school time. Using data from Sky and Telescope, the times of maximum displacement from Jupiter of the four satellites were estimated. It was discovered that within the time period available for investigation, only Io at its maximum displacement could be photographed at night.

Europa's orbit was used to verify the results obtained as though the maximum displacement could not be taken, photos were taken near the estimated time of maximum displacement. The period of Callisto and Ganymede's orbits were difficult to capture within the time frame available.

Through taking photos of Jupiter, the pixel displacement of the satellites from Jupiter can be found and plotted against time. Using trial and error a sine curve matching the points was found. Hence the period and the radius of orbit can be calculated.

To convert the pixel radius into the true radius, various data items are needed:

1) The conversion from pixels to an angular distance by the camera
2) The magnification of the telescope
3) The distance from Earth to Jupiter

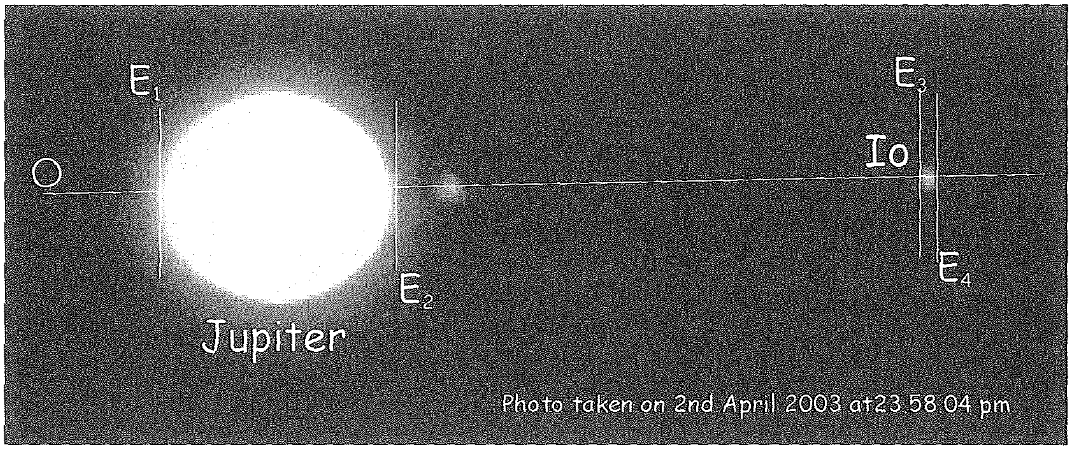
Section A: Photography

Method

Photos of Jupiter were taken from the 2nd April to the 11th April from 10pm to 2am, every half hour using a telescope (MEADE LX200 with a 200mm aperture) and a camera (Olympus C900 ZOOM). It was limited to these 4 hours due to the inability to see Jupiter. Earlier than 10pm, the telescope pointed at the roof and later than 2am, the buildings and trees blocked Jupiter.

Every half hour, 6 photos were taken at different settings but all at full zoom. Two photos were under exposed (less light let in), two photos were taken with normal settings and two photos were over exposed (more light let in). The over exposed photos allow the satellites to be seen more clearly as they were not easily detected due to their small size. However, the over exposure makes the edges of Jupiter blurred so the image was larger than it should have been.
The 10 second delay timer was used to minimize camera shake, as a cardboard ring held the camera to the telescope
The photos were taken using a digital camera for various reasons. It was available; less time was needed to process photos as no developing is needed minimizing costs. Furthermore, only the magnification of the camera and the telescope are taken into account instead of also the magnification of the photo developer, from film to photo. Furthermore, using the Microsoft Photo Editor, the pixel length of the orbit was calculated.

Sample Photo



The coordinates of OE_1, OE_2, OE_3 and OE_4 were found using Microsoft Photo Editor. Using Pythagoras' Theorem, $a^2 = b^2 + c^2$, the diameter of Jupiter (D_j) and Io(D_i) and the distance between E_2 and E_3(d) was found.

The radius of Io's orbit (r) was found using $r = d + \frac{D_i + D_j}{2}$.

Sample Calculation 1: The radius of Io's Orbit

[Note: Each coordinate has an error of ± 1 pixel]
E_1 = (550, 425)
E_2 = (620, 423)
E_3 = (751, 420)
E_4 = (755, 420)
[Note: Both D_j, D_i and D have an error of ± 8 pixels]

$D_j = \sqrt{(550-620)^2+(425-423)^2} = 70.03$ pixels

$D_i = \sqrt{(755-751)^2+(420-420)^2} = 4$ pixels

d = $\sqrt{(751-620)^2+(420-423)^2} = 131.03$ pixels

r $= 131.03 + \frac{70.03+4}{2} = 168.05$ pixels

Error of r = $8 + \frac{8+8}{70.03+4} \times \frac{1}{2} \times \frac{70.03+4}{2} = 8 + \frac{16}{4} = 12$ [constant error for r]

Results: Focus on Io

372 usable photos were taken but due to the large volume of data obtained, a small sampling of the data is provided in Table 1.

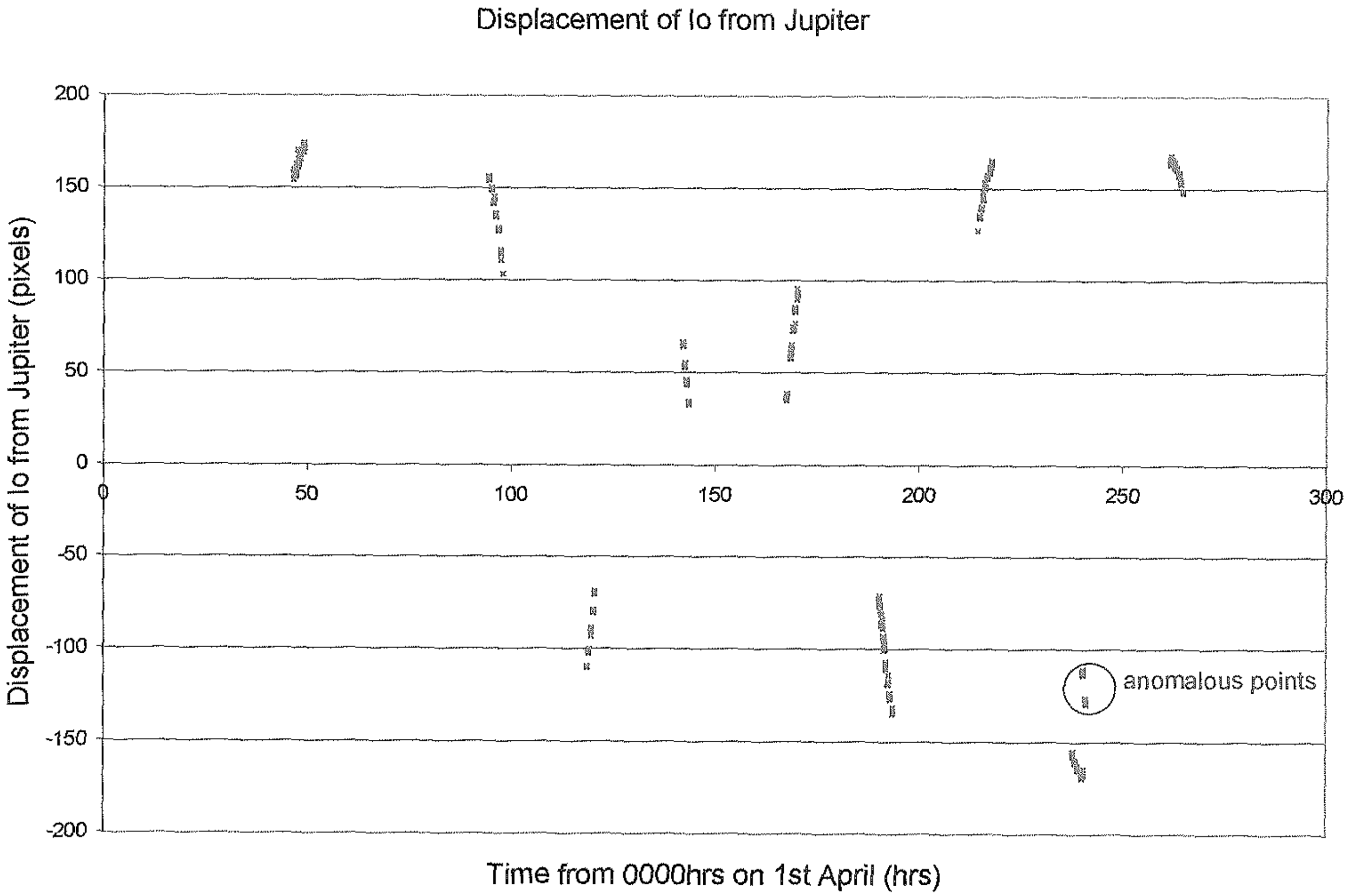
A time-displacement graph was plotted (see Graph 1) and using trial and error a sine curve fitting the points was found, in the form y = Asin(Bx+C) where y is the displacement, and x is the time. The values of A, B and C were adjusted so that the curve fitted the points. A is the displacement of Io from Jupiter, B is the period of Io's orbit and C is a figure to shift the curve along the x-axis.

Table 1: Pixel Radius of Io's Orbit

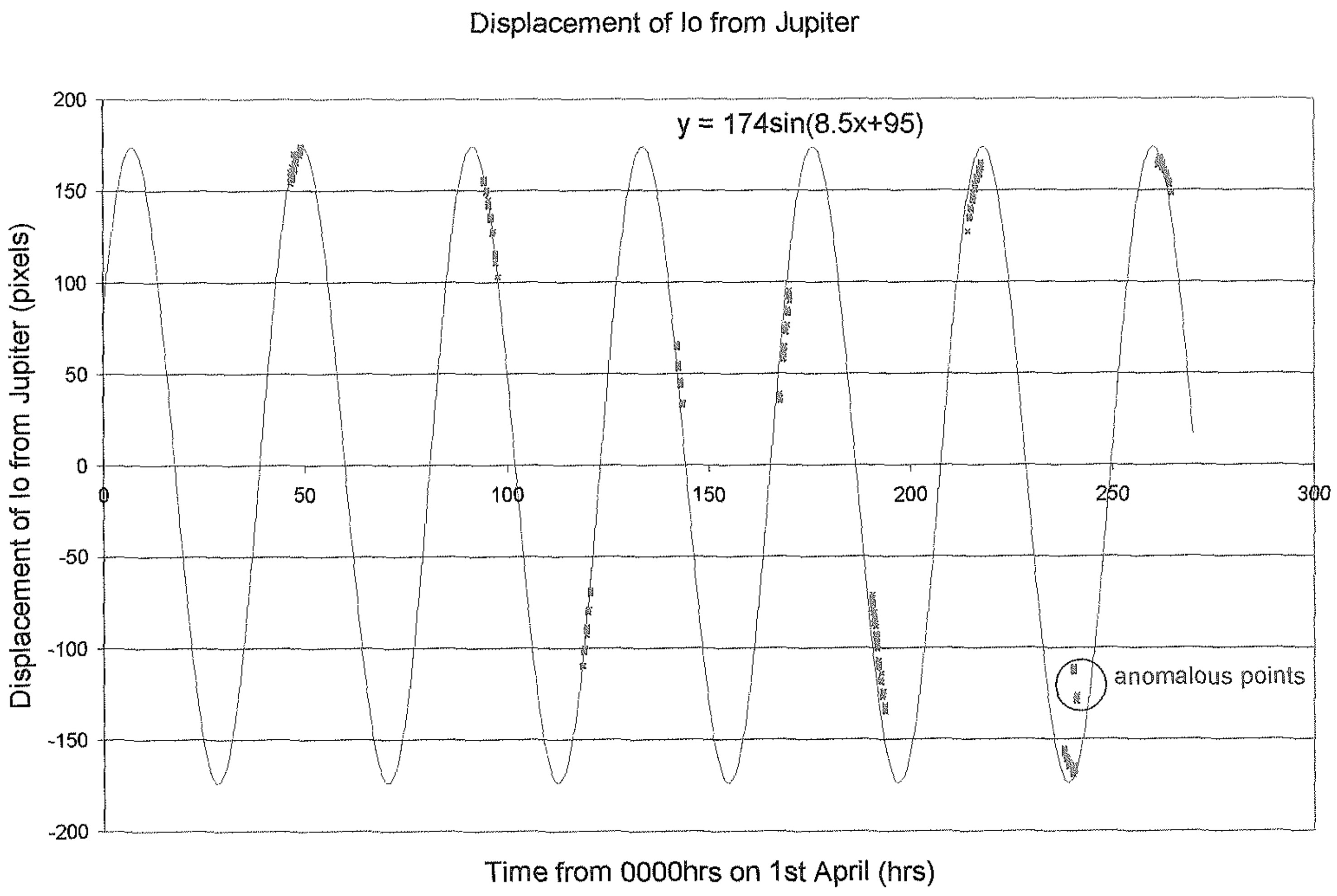
Date	Time	Time from 1st April 0000hrs (hrs)	Jupiter					Io					Distance from edge of Jupiter to Io ±8	Displacement of Io from Jupiter (pixels) ±12
			Left Edge ±1		Right Edge ±1		Diameter (in pixels) ±8	Left Edge ±1		Right Edge ±1		Diameter (in pixels) ±8		
02-Apr	22.24.52	46.4144	663	544	721	514	65	829	465	833	461	6	119	154
02-Apr	22.27.38	46.4606	681	541	737	514	62	853	462	856	461	3	127	160
02-Apr	22.57.00	46.9500	667	489	735	466	72	849	433	853	432	4	119	157
02-Apr	22.59.16	46.9878	610	501	677	483	69	794	454	798	452	4	121	157
02-Apr	23.04.30	47.0750	615	518	681	500	68	798	472	804	470	6	120	158
02-Apr	23.06.48	47.1133	628	516	697	500	71	814	477	817	476	3	119	156
02-Apr	23.29.30	47.4917	532	550	606	540	75	735	522	739	521	4	130	170
02-Apr	23.31.58	47.5328	574	534	651	526	77	770	514	775	515	5	120	161
02-Apr	23.59.06	47.9850	570	435	639	434	69	768	431	771	431	3	129	165
02-Apr	00.02.02	48.0339	600	399	661	397	61	799	396	801	394	3	138	170
02-Apr	00.52.36	48.8767	642	453	709	466	68	842	491	847	492	5	135	172
02-Apr	00.54.56	48.9156	660	483	722	495	63	857	519	861	520	4	137	171
02-Apr	00.56.10	48.9361	663	481	727	494	65	860	521	865	523	5	136	171
02-Apr	01.00.38	49.0106	656	423	717	433	62	854	459	859	460	5	139	173
03-Apr	01.04.46	49.0794	662	457	736	470	75	867	495	872	495	5	133	173
03-Apr	01.05.26	49.0906	619	452	692	466	74	824	493	828	494	4	135	174
03-Apr	22.20.54	94.3483	662	508	718	484	61	832	438	834	435	4	123	155
03-Apr	22.22.44	94.3789	600	467	653	446	57	769	402	772	400	4	124	154
03-Apr	22.58.22	94.9728	669	467	727	453	60	841	428	845	428	4	117	149
03-Apr	22.00.40	95.0111	689	498	747	486	59	863	462	865	463	2	118	149

The red row is the time of maximum displacement according to the results.

Graph 1: Io's Displacement from Jupiter



Graph 2: Io's Displacement from Jupiter with a sine curve fitted to the points



Graph 3: Io's Displacement from Jupiter in pixels with the error bars, minimum and maximum possible sine curves

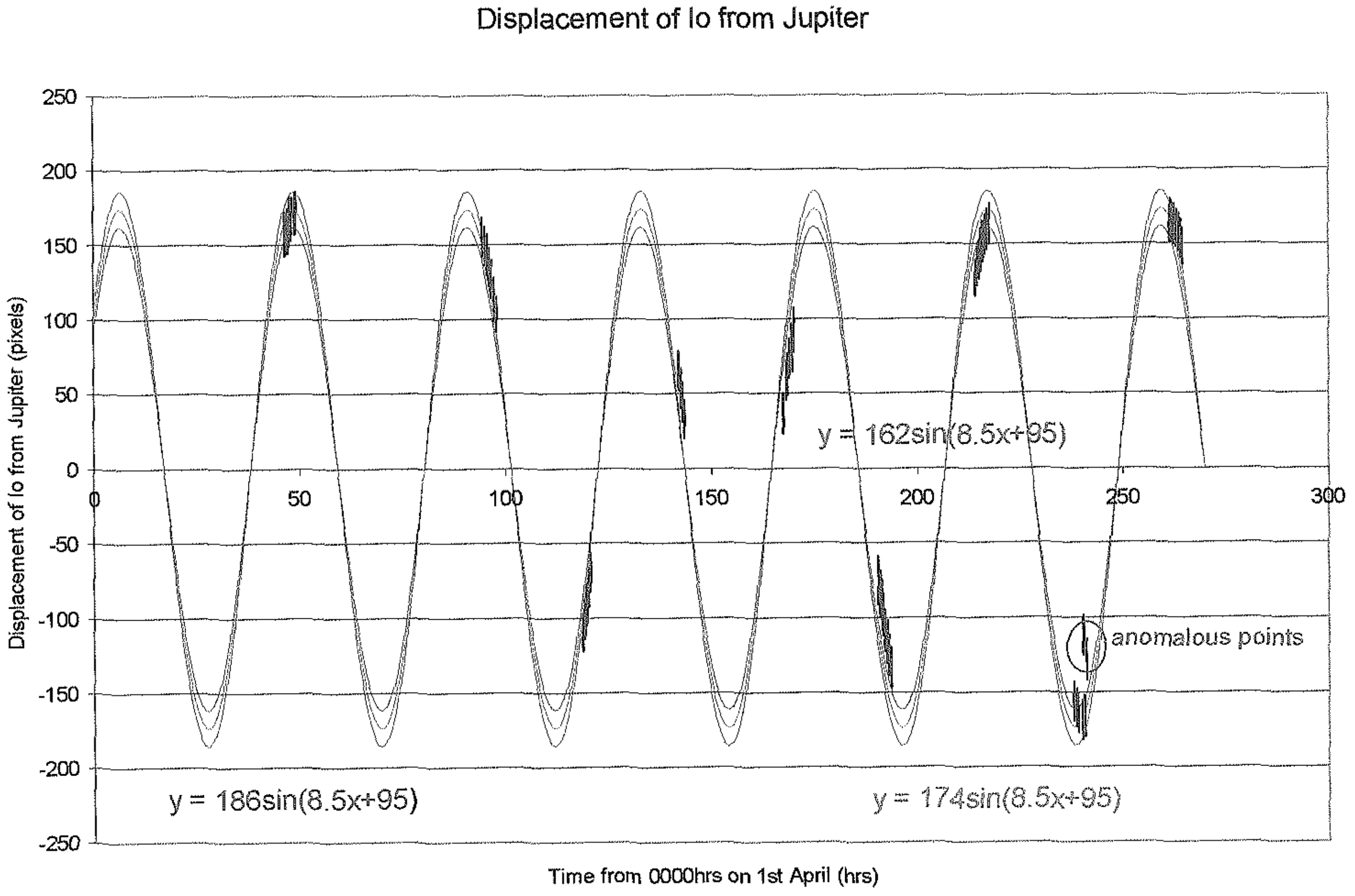


Table 2: Sine curve equations

	Sine curve equation	Maximum displacement (i.e. Radius of orbit)
Min	y = 162sin(8.5x+95)	162
	y = 174sin(8.5x+95)	174
Max	y = 186sin(8.5x+95)	186

The radius of the orbit of Io is 174 pixels ± 12.

Anomalies

Looking at Graph 1, there are several anomalous points which are shown in table 3.
The diameter of Jupiter (which is a constant) at the anomalous points is between 39-48 pixels, while the diameter of Jupiter for other points was 54 pixels or higher.
Hence it can be concluded that these anomalous points were due to the camera not on full zoom.

Table 3: Anomalous results

Date	Time	Time from 1st April 0000hrs	Jupiter					Io					Distance from edge of Jupiter to Io ±8	Radius of the orbit (pixels) ± 12	Displacement of Io from Jupiter (pixels) ± 12
			Left Edge ±1		Right Edge ±1		Diameter (in pixels) ±8	Left Edge ±1		Right Edge ±1		Diameter (in pixels) ±8			
11-Apr	00:02:38	240.0439	646	215	701	225	56	504	200	508	201	4	139	169	-169
11-Apr	00:03:28	240.0578	610	206	664	215	55	467	189	471	191	4	140	169	-169
11-Apr	00:16:28	240.2744	596	294	635	302	40	504	282	506	282	2	90.8	112	-112
11-Apr	00:17:16	240.2878	608	296	647	299	39	514	280	519	279	5	90.6	113	-113
11-Apr	00:18:12	240.3033	615	297	656	303	41	522	285	527	286	5	88.7	112	-112
11-Apr	00:19:00	240.3167	612	300	652	305	40	518	287	523	288	5	89.8	113	-113
11-Apr	00:19:52	240.3311	622	311	662	316	40	528	295	532	297	4	91.1	113	-113
11-Apr	00:21:04	240.3511	586	451	626	457	40	494	440	498	440	4	88.7	111	-111
11-Apr	00:31:36	240.5267	596	419	655	423	59	457	398	463	398	6	135	167	-167
11-Apr	00:32:22	240.5394	607	414	661	420	54	467	395	470	396	3	138	167	-167
11-Apr	00:33:08	240.5522	631	412	686	419	55	491	392	494	394	4	138	168	-168
11-Apr	01:01:26	241.0239	565	429	612	436	48	461	409	466	410	5	101	127	-127
11-Apr	01:02:14	241.0372	581	424	623	432	43	474	404	478	403	4	105	129	-129
11-Apr	01:03:26	241.0572	582	406	626	414	45	476	385	478	385	2	106	129	-129

The data in red are anomalous points.

The period of Io's orbit

It is possible to calculate the orbit, by solving the sine curve for $y = 0$.
$y = 175\sin(8.5x + 95)$
$0 = \sin(8.5x + 95)$
$8.5x + 95 = \sin^{-1} 0 = 0$ hours, 180 hours, 360 hours

Hence $x = -10.8$ hours, 10 hours, 31.2 hours
The period of the orbit is $31.2 - (-10.8) = 42$ hours.

Conclusion

The radius of Io's orbit around Jupiter is 174 pixels ± 12 with a period of 42 hours. According to NASA, the period of Io's orbit is 1.769138 days[4] which is equal to 42.5 hours, meaning that the period of Io's orbit obtained is close to the accepted value. The anomalous results are ignored for the rest of the investigation as the camera was not in full zoom for them.

Results: Focus on Europa

The photos were analysed to create another result table focusing on Europa. Table 4 provides a small sampling of the data. The points are plotted onto a time-displacement graph and a sine curve is fitted to them.

---

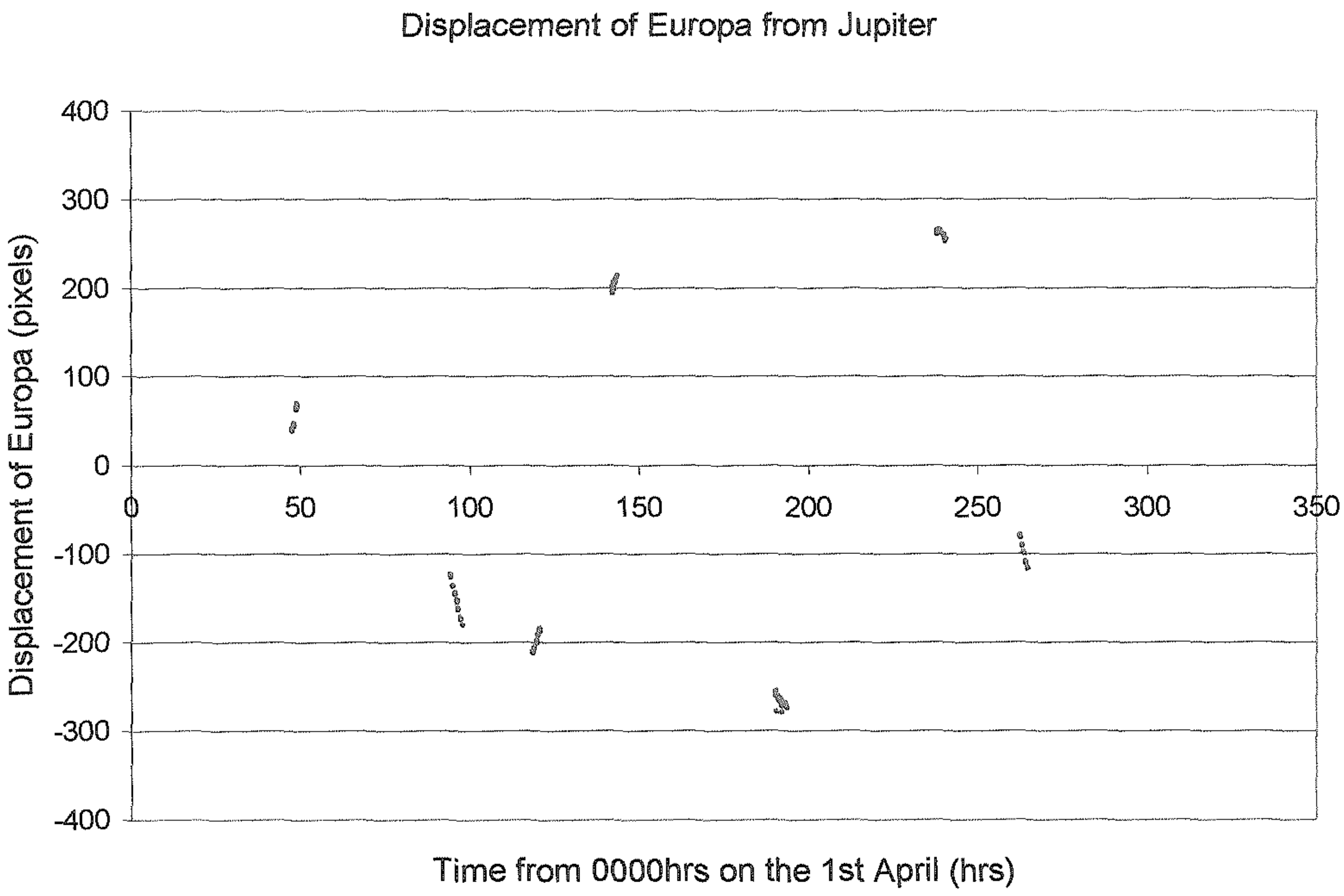
[4] NASA - Gallileo: Journey to Jupiter
Io Fact Sheet; Shannon McConnell.
Available from the World Wide Web:<http://www.jpl.nasa.gov/galileo/io/fact.html>

Table 4: Pixel Radius of Europa's Orbit

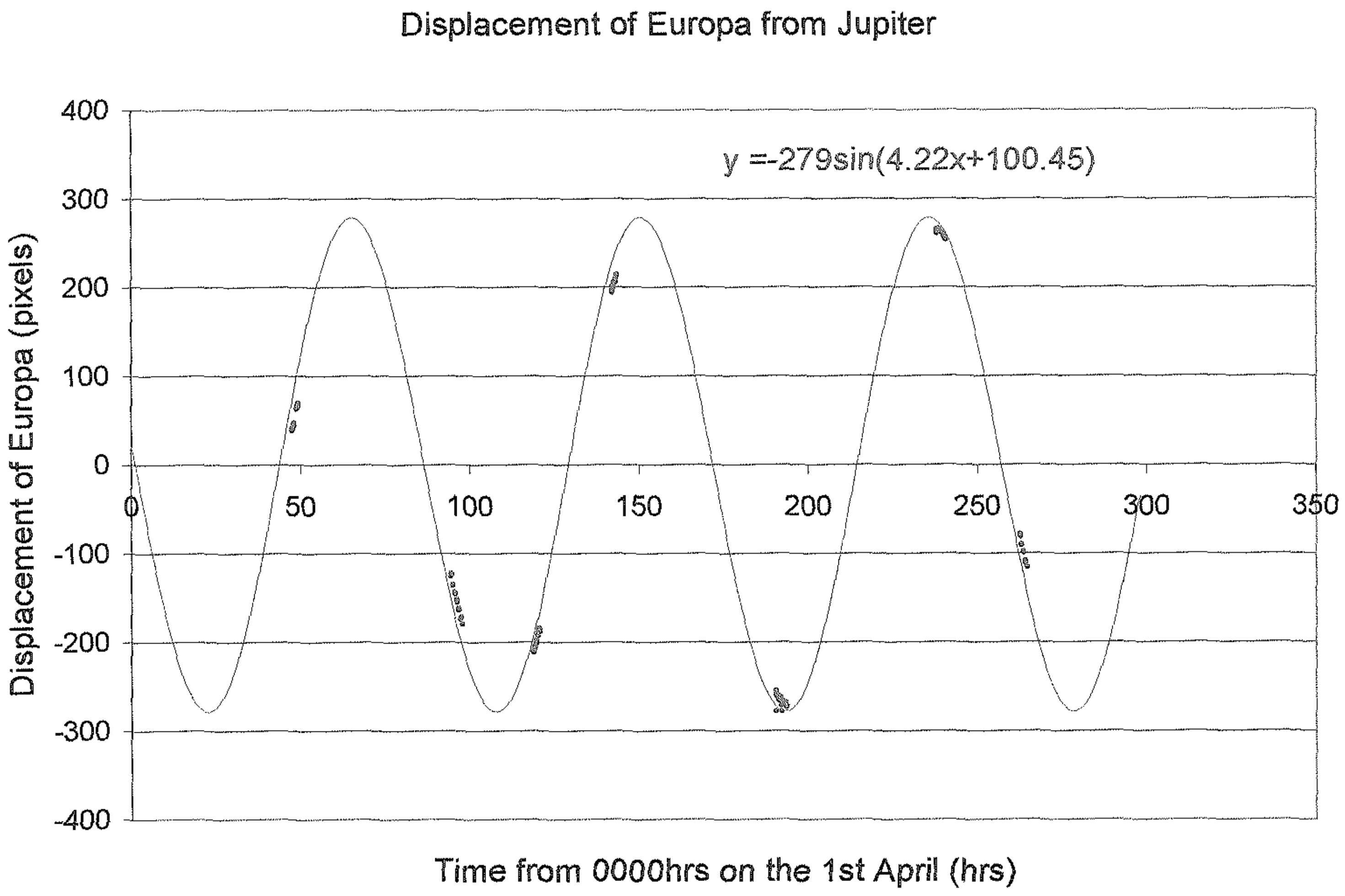
Date	Time	Time from 1st April 0000hrs	Jupiter					Europa					Distance from edge of Jupiter to Europa ± 8	Distance from center of Jupiter to center of Europa (pixels) ± 12	Displacement of Europa from Jupiter (pixels) ± 12
			Left Edge ± 1		Right Edge ± 1		Diameter (pixels) ±8	Left Edge ± 1		Right Edge ± 1		Diameter (in pixels) ± 8			
08-Apr	23:17:44	191.2956	550	433	611	431	61	314	439	319	438	5	231	264	-264
08-Apr	23:19:46	191.3294	575	399	635	398	60	339	403	342	404	3	233	265	-265
08-Apr	23:21:44	191.3622	535	407	596	406	61	300	410	306	410	6	229	263	-263
08-Apr	23:23:46	191.3961	553	390	619	393	66	321	394	326	394	5	227	263	-263
08-Apr	23:34:04	191.5678	583	363	644	369	61	342	354	351	354	9	232	267	-267
09-Apr	00:00:16	192.0044	532	335	591	341	59	289	309	286	309	3	247	279	-279
09-Apr	00:01:48	192.0300	579	309	641	315	62	343	284	348	284	5	232	266	-266
09-Apr	00:31:24	192.5233	559	388	611	399	53	317	339	321	340	4	243	271	-271
09-Apr	00:32:52	192.5478	577	352	637	365	61	341	303	345	304	4	237	270	-270
09-Apr	01:00:54	193.0150	619	395	682	405	64	384	348	389	350	5	234	269	-269
09-Apr	01:30:50	193.5139	562	442	612	454	51	318	393	323	392	5	244	272	-272
10-Apr	21:57:34	237.9594	693	433	747	409	59	961	324	964	322	4	230	262	262
10-Apr	21:59:12	237.9867	671	416	728	393	61	943	308	949	305	7	231	265	265
10-Apr	22:31:16	238.5211	748	363	811	348	65	1032	299	1038	298	6	226	262	262
10-Apr	22:33:34	238.5594	725	345	786	330	63	1013	290	1018	289	5	230	264	264
10-Apr	23:01:32	239.0256	681	291	738	285	57	970	267	973	266	3	233	263	263
11-Apr	00:32:40	240.5444	626	411	679	417	53	905	446	907	447	2	228	256	256
11-Apr	22:30:02	262.5006	641	404	695	384	58	590	416	595	416	5	48	79	-79
11-Apr	22:31:36	262.5267	654	407	711	390	59	603	421	607	418	5	48	81	-81
11-Apr	23:04:30	263.0750	591	410	647	401	57	526	422	532	420	6	60	91	-91
11-Apr	23:06:02	263.1006	603	387	660	376	58	541	396	544	394	4	59	90	-90
11-Apr	23:33:52	263.5644	546	308	606	303	60	474	313	479	311	5	67	100	-100

The red row is the time of maximum displacement according to the results.

Graph 4: *Europa's Displacement from Jupiter*



Graph 5: Europa's displacement from Jupiter with a sine curve fitted to the points



Graph 6: Europa's Displacement from Jupiter in pixels including error bars, maximum and minimum possible sine curves

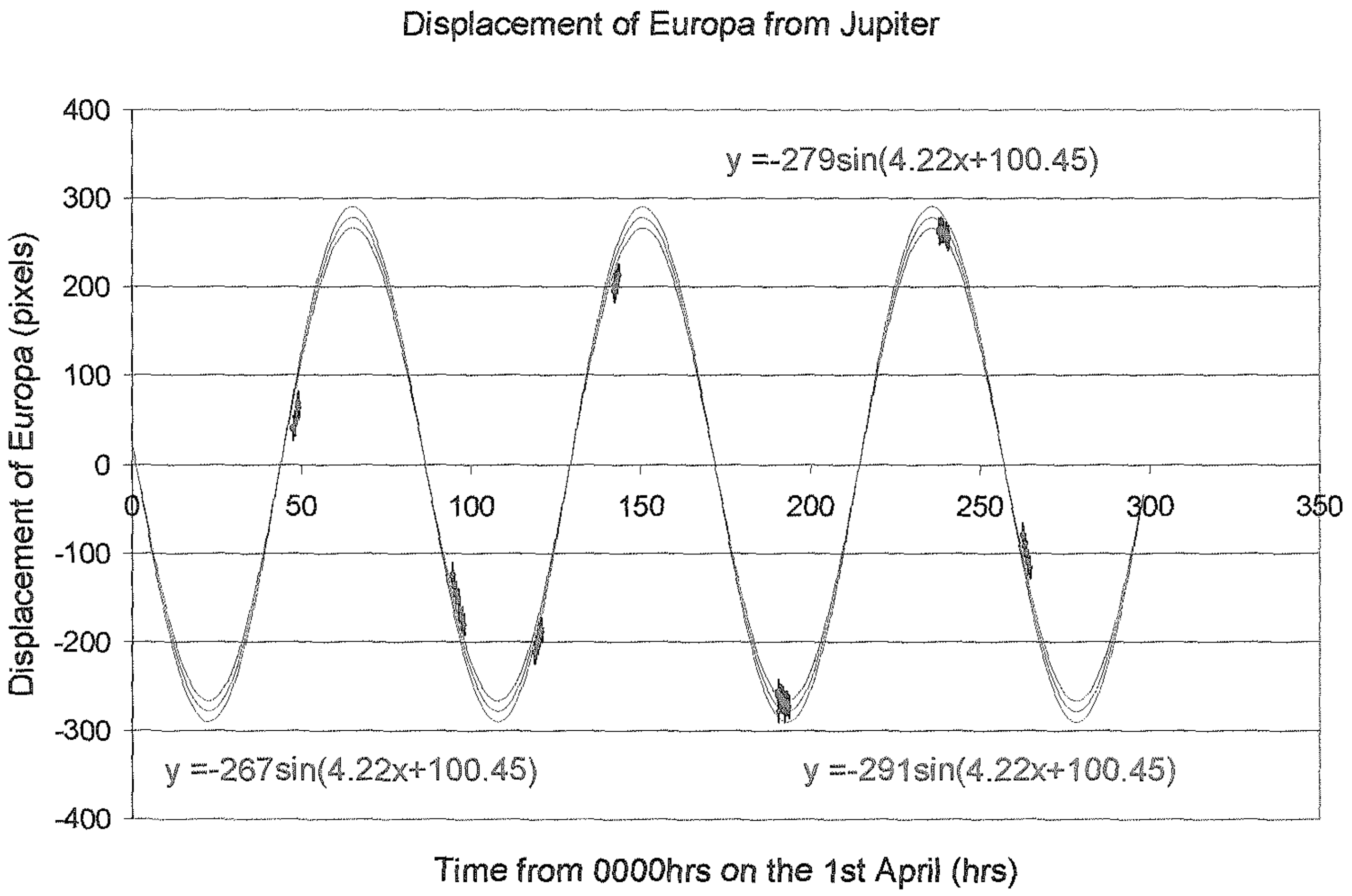


Table 5: Sine curve equations

	Sine curve equation	Maximum displacement (i.e. Radius of orbit)
Min	$y = -267\sin(4.22x+100.45)$	-267
	$y = -279\sin(4.22x+100.45)$	-279
Max	$y = -291\sin(4.22x+100.45)$	-291

The radius of Europa's orbit is 279 pixels ± 12.

The period of Europa's orbit

Again, it is possible to calculate the orbit, by solving for $y = 0$.
$y = -279\sin(4.22x + 100.45)$
$0 = \sin(4.22x + 100.45)$
$4.22x + 100.45 = \sin^{-1} 0 = 0$ hours, 180 hours, 360 hours

Hence $x = -23.8$ hours, 18.9 hours, 61.5 hours
The period of the orbit is 61.5 – (-23.8) = 85.3 hours.

Conclusion

The radius of Europa's orbit is 279 pixels ± 12 and the orbit has a period of 85.3 hours. NASA states that the orbital period of Europa is 3.551181[5] days which is equal to 85.2 hours.
While the period is close to the real value, the radius of the orbit may not be because the main focus of the investigation was Io and the time of maximum displacement of Europa according to my results was earlier than the estimated time. Therefore, the value of the radius of the orbit is not the true maximum displacement but in fact slightly smaller but as the photo was taken at a time close to the estimated time of Europa's maximum displacement, the value is approximated as the maximum.

---

[5] NASA - Gallileo: Journey to Jupiter
Europa Fact Sheet; Shannon McConnell.
Available from the World Wide Web:<http://www.jpl.nasa.gov/galileo/europa/>

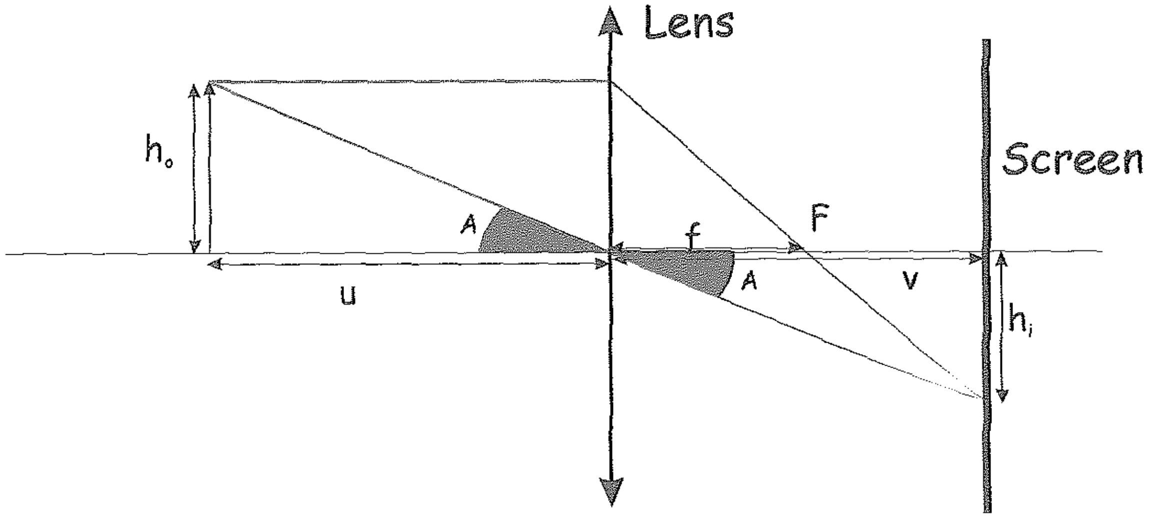
Section B: Optics of the camera and the telescope

To calculate the angle corresponding to one pixel on the photo, within the camera and within the telescope.

Optics of the camera

To calculate the angle corresponding to one pixel on the photos within the camera, a photo of a nearby object was taken so that real distances could be measured.

Diagram 1: Optics of the Camera



Angle A is calculated and using trigonometry, $\tan A = \frac{h_o}{u}$, hence $A = \tan^{-1}\left(\frac{h_o}{u}\right)$.

However, what is required is the angle per pixel, so we divide A by h_i.
So, a photo of a TV was taken, the distance u, h_o and h_i were measured.

Results: The angle per pixel in the camera

u = 3800mm ± 1 [% error = 0.03%]
h_o = 442mm ± 1 [% error = 0.22%]
h_i = 406 pixels ± 1 [% error = 0.25%]
$\frac{h_o}{u} = \frac{442mm}{3800mm} = 0.116$ [% error = 0.25%]

Table 6: Calculating A

$\frac{h_o}{u}$	$A = \tan^{-1}\left(\frac{h_o}{u}\right)$ (radians)
0.116 - 0.25%	0.1152
0.116	0.1155
0.116 + 0.25%	0.1158

It is seen that the error is ± 0.003 (2.6%).

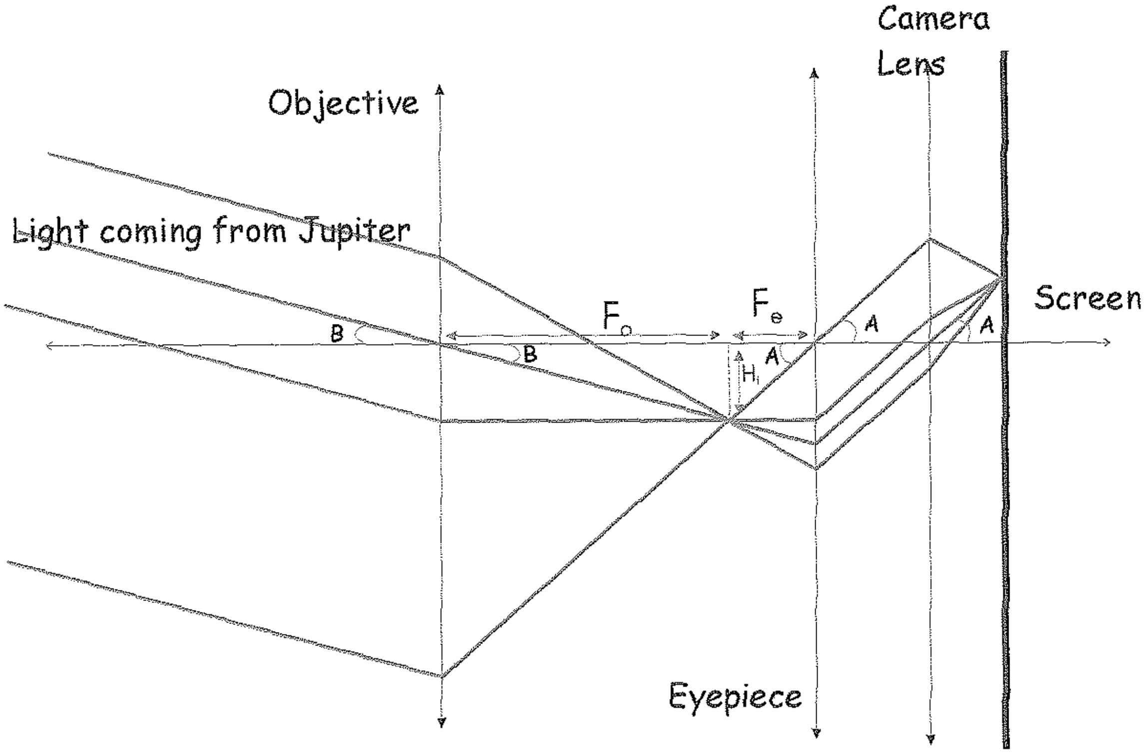
$\frac{A}{h_i} = \frac{0.1155rad}{406pixels} = 0.00029rad$ [% error = 2.85%]

[Note: radians are used as $\theta \approx \tan\theta$ when θ is a small angle in radians - this approximation will be used for the investigation as the angles are small]

Optics of the Telescope

Using an optic diagram that combines the effect of the telescope's magnification and the camera's conversion from pixels to an angular distance, the angle the light enters the telescope is calculated.

Diagram 2: Optics of the Telescope and the Camera



Light coming from Jupiter, due to the distances involved are taken as parallel beams of light. The light enters the objective lens refracts to meet at the focal point of the objective lens and the eyepiece. The rays of light refracted by the eyepiece produce a virtual image at infinity and the refracted rays are parallel rays. These parallel rays enter the camera lens and focus at a point on the screen to produce an image.

From the diagram, $A = \frac{H_i}{F_e}$ and $B = \frac{H_i}{F_o}$. By rearranging the formulae and substitution, we get $AF_e = BF_o$ and that $B = \frac{AF_e}{F_o}$.

Results: The angle per pixel through the telescope and camera

F_e = 26mm
F_o = 2000mm
$B = \frac{0.00029 \times 26}{2000} = 3.77 \times 10^{-6} rad$ [% error = 2.85%]

Conversion of Io's radius from pixels into radians

The result of the radius of Io's orbit from Section A is used to calculate the angular radius.
r_p = 174 pixels ± 12 [% error = 6.9%]

So angular radius = $r_p \times B = 3.77 \times 10^{-6} rad \times 174 pixels = 6.56 \times 10^{-4} rad$
[% error =9.75%]

Conversion of Europa's radius from pixels into radians

The result of the radius of Europa's orbit is used to calculate the angular radius.
r_p = 279 pixels ± 12 [% error = 4.3%]
So angular radius = $r_p \times B = 3.77 \times 10^{-6} rad \times 279 pixels = 0.0012 rad$
[% error = 7.15%]

Section C: Distance from Earth to Jupiter

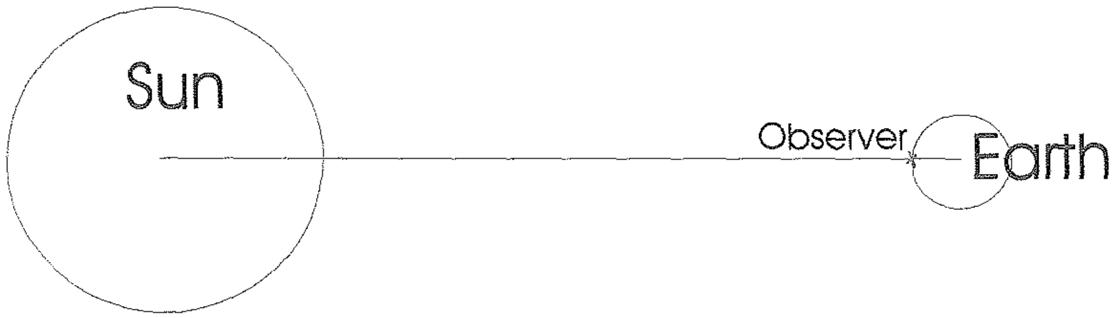
Before the mass of Jupiter can be calculated we must calculate the distance from Earth to Jupiter.
To do this, we need to know

1) Time of the Sun's Zenith (Time 1)
2) Time of Jupiter's Zenith (Time 2)
3) Angle the Earth rotates between Time 1 and 2
4) Distance from Sun to Earth
5) Distance from Sun to Jupiter

1. The time of the Sun's Zenith

The Sun's zenith is when a straight line goes through the center of the Sun to the center of Earth passing through the point of the observer.

Diagram 3: Sun to Earth



This time is midway between the sunrise and sunset time, which was obtained from the astronomical program Redshift 3, so it can be calculated for the relevant days

Results: Time of the Sun's Zenith

Sample Calculation 2: Time of the Sun's Zenith
2nd April 2003
Sunrise: 7:06 ± 30s
Sunset: 19:14 ± 30s

Time 1: $\frac{7\frac{1}{10}+19\frac{7}{30}}{2}=13\frac{1}{6}$=13:10 ± 60s

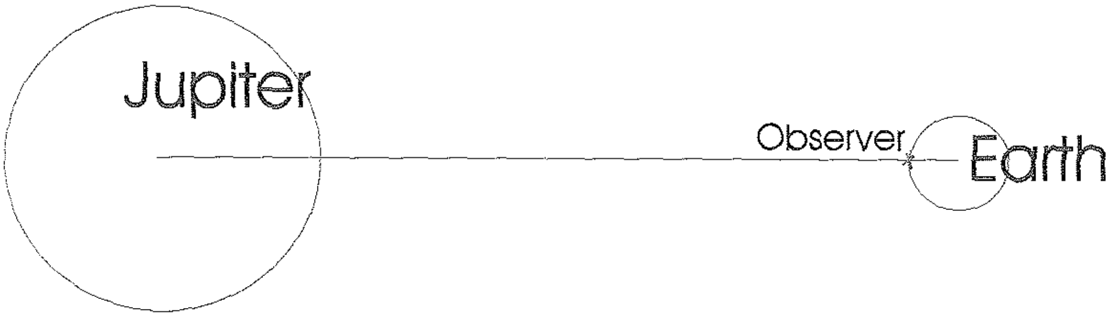
Table 7: Time of the Sun's Zenith[6]

Date	Sunrise ± 30s	Sunset ± 30s	Time 1 ± 60s
02-Apr-03	7:06	19:14	13:10
03-Apr-03	7:06	19:13	13:09
04-Apr-03	7:05	19:13	13:09
05-Apr-03	7:05	19:13	13:09
06-Apr-03	7:05	19:13	13:09
07-Apr-03	7:04	19:12	13:08
08-Apr-03	7:04	19:12	13:08
09-Apr-03	7:04	19:12	13:08
10-Apr-03	7:03	19:12	13:08
11-Apr-03	7:03	19:11	13:07

2. Time of Jupiter's Zenith

Likewise with the Sun, the time of Jupiter's Zenith is when a straight line from the centre of Jupiter to the centre of Earth goes through the point of the observer.

Diagram 4: Earth to Jupiter



Again, this time is midway between the Jupiter rise and Jupiter set time, obtained from the astronomical program Redshift 3, so it can be calculated for the relevant days.

Results: Time of Jupiter's Zenith

Sample Calculation 3: Time of Jupiter's Zenith

2nd April 2003

Jupiter rise: 15:04 ± 30s

Jupiter set: 3:14 ± 30s [on the 3rd April]

Time 2: $\frac{15\frac{1}{15}+3\frac{7}{30}+24}{2} = 21\frac{3}{20}$ =21:09 ± 60s

---

[6] **Redshift 3**; United Soft Media; November 1998

Table 8: Time of Jupiter's Zenith[7]

Date	Jupiter rise ± 30s	Jupiter set ± 30s	Time 2 ± 60s
02-Apr-03	15:04	3:14	21:09
03-Apr-03	15:00	3:10	21:05
04-Apr-03	14:56	3:06	21:01
05-Apr-03	14:52	3:03	20:57
06-Apr-03	14:48	2:59	20:53
07-Apr-03	14:44	2:56	20:50
08-Apr-03	14:40	2:51	20:46
09-Apr-03	14:36	2:47	20:42
10-Apr-03	14:32	2:43	20:38
11-Apr-03	14:29	2:39	20:34

Conclusion

At the time of Jupiter's zenith, the figure calculated is not exactly right, as the Earth and Jupiter orbit around the Sun and are constantly changing position in relation to each other. The time when of Jupiter's zenith is in fact two minutes after the time calculated due to the movement of the two planets. Therefore, two minutes is added onto the error of the Jupiter zenith making the error ± 180s.

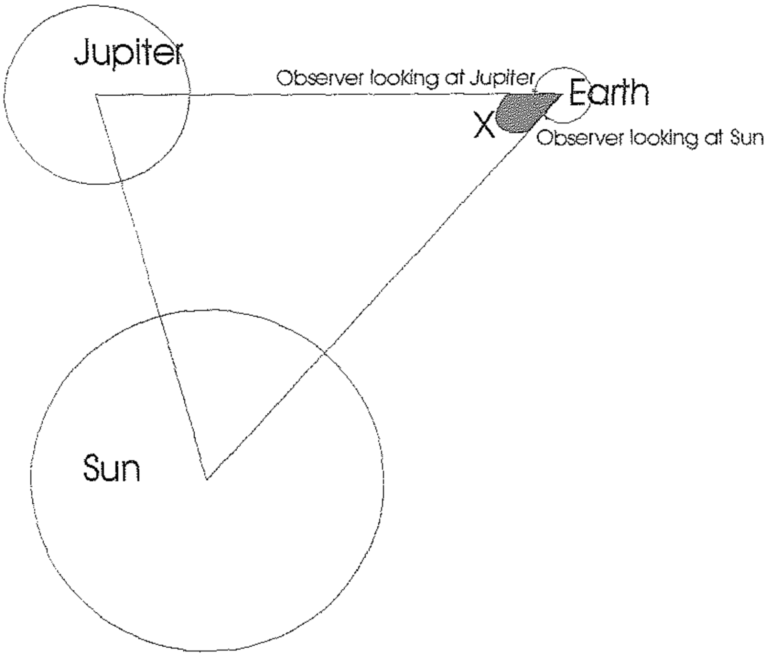
---

[7] Redshift 3; United Soft Media; November 1998

3. Angle the Earth rotates between Time 1 and 2

Diagram 3 and 4 are put together to produce diagram 5 below.

Diagram 5: Earth, Jupiter and the Sun



X is the angle the Earth rotates between Time 1 and 2. As the Earth rotates 0.25 degrees a minute, the time difference between Time 1 and 2 can be found, and hence angle X can be calculated.

Results: Angle of rotation between Time 1 and 2

Sample Calculation 4: Angle of rotation between Time 1 and 2
2nd April 2003
Time 1: 13:10 ± 60s
Time 2: 21:09 ± 180s
Time between Time 1 and Time 2: $21\frac{3}{20}-13\frac{1}{6}=7\frac{59}{60}$ = 7:59 ± 240s [% Error = 0.84%]
X: $7\frac{59}{60}\times 15=119.75^\circ \pm 1$ [% error = 0.84%]
(Constant error for X)

Table 9: Angle of rotation between Time 1 and 2

Date	Time 1 ± 60s	Time 2 ± 180s	Time between Time 1 and Time 2 ± 240s	% Error	X° ± 1
02-Apr-03	13:10	21:09	7:59	0.84%	119.750
03-Apr-03	13:09	21:05	7:56	0.84%	118.875
04-Apr-03	13:09	21:01	7:52	0.85%	118.000
05-Apr-03	13:09	20:57	7:49	0.85%	117.125
06-Apr-03	13:09	20:53	7:45	0.86%	116.125
07-Apr-03	13:08	20:50	7:42	0.87%	115.375
08-Apr-03	13:08	20:46	7:38	0.87%	114.375
09-Apr-03	13:08	20:42	7:34	0.88%	113.375
10-Apr-03	13:08	20:38	7:30	0.89%	112.500
11-Apr-03	13:07	20:34	7:27	0.89%	111.750

4. Distance from the Sun to Earth

The Earth-Sun ephemeris obtained from NASA gives the distance from the Sun to Earth over the relevant days.

Table 10: Data from the Earth-Sun Ephemeris[8]

Date (0 TDT)	Distance from Sun to Earth (a.u.)
01-Apr-03	0.999128
03-Apr-03	0.999703
05-Apr-03	1.000274
07-Apr-03	1.00084
09-Apr-03	1.001401
11-Apr-03	1.00196
13-Apr-03	1.002517

Terrestrial Dynamical Time (TDT) is calculated using the formula UTC + dT, where UTF is the Coordinated Universal time and dT is "a slowly growing interval of time [dT] that now amounts to a little more than a minute."[9] This means each year dT is a different value and according to NASA, where in 2003 dT = 64.6s.[10] This value is small on the scale of data calculated, so taking this into account would have little impact on the calculations.

---

[8] NASA/Goddard Space Flight Center
Geocentric Ephemeris for the Sun; Fred Espenak.
Available from the World Wide Web:<http://sunearth.gsfc.nasa.gov/eclipse/TYPE/TYPE.html>

[9] **Delta T: Approximate Algorithms for Historical Periods**; Robert H. van Gent.
Available from the World Wide Web:<http://www.phys.uu.nl/~vgent/astro/deltatime.htm>

[10] NASA/Goddard Space Flight Center
Delta T and Universal Time; Fred Espenak.
Available from the World Wide Web:<http://sunearth.gsfc.nasa.gov/eclipse/SEhelp/deltaT.html>

Hence conversion from TDT to GMT +0800hrs (Singapore Time) and an equation showing the relationship between time and distance of the Earth from the Sun are needed.

Results: Distance from the Sun to Earth

Sample Calculation 5: Conversion from TDT to GMT +0800hrs

1st April 2003 0000hrs TDT

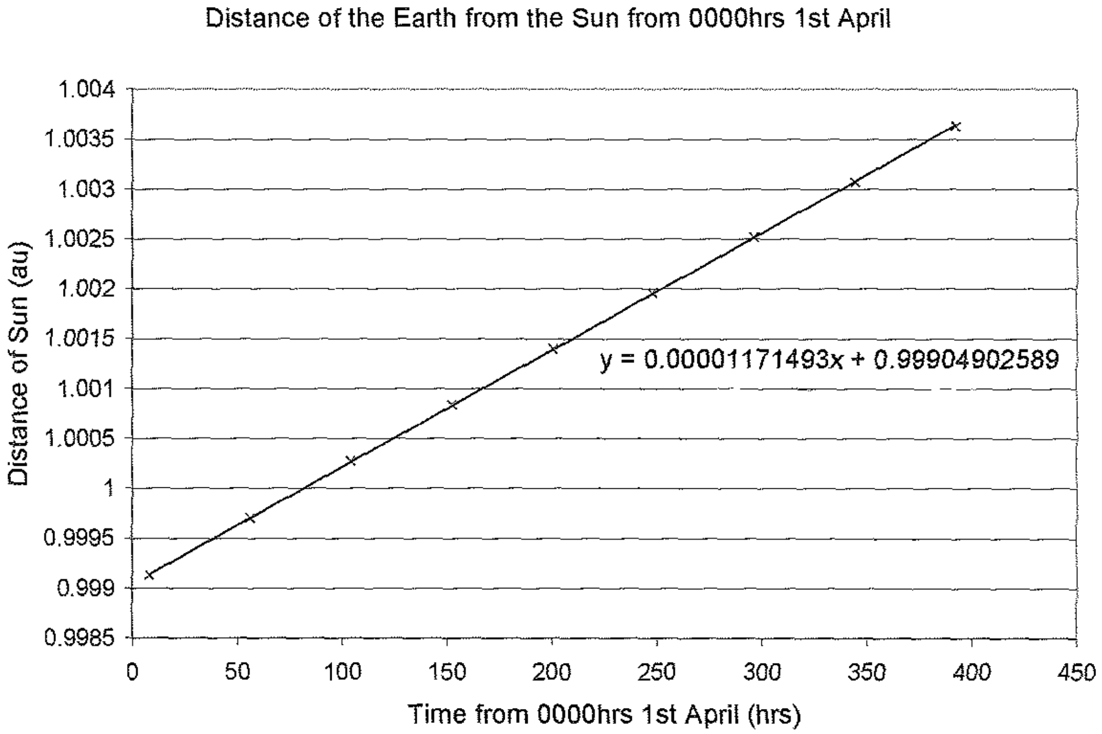
GMT +0800hrs: $\frac{64.6s}{3600s} + 8hrs + 24hrs \times 0 \text{ days from } 1^{st} \text{ April} = 8.018hrs$

Table 11: Conversion from TDT to GMT +0800hrs

Time from the 1st April 0000hrs (hrs)	Distance from Sun to Earth (au)
8.017944	0.999128
56.01794	0.999703
104.0179	1.000274
152.0179	1.00084
200.0179	1.001401
248.0179	1.00196
296.0179	1.002517

It is seen that up to the 4th significant figure, the data is almost the same over the period of time rounding up to 1 au. This is an error of about 0.01%, which is fairly insignificant but as some errors are along that scale (0.13%), this error may make a difference.

Graph 7: Distance of the Earth from the Sun



The equation is y = 0.00001171493x + 0.99904902589, where y is the distance of Earth from the Sun and x is the time from 0000hrs on the 1st of April.

Sample Calculation 6: Distance the Earth is from the Sun at the times of the Sun Zenith

2nd April 2003
Time: 13:10

Time from 1st April 0000hrs: $13\frac{1}{6}hrs + 24hrs \times 1 \text{ day from } 1^{st} \text{ April} = 37\frac{1}{6}hrs$

[% error = 0.13%] (constant % error for Time 1)

Distance: $0.00001171493 \times 37\frac{1}{6}hrs + 0.99904902589 = 0.999578\text{au} \pm 0.0013$

[% error = 0.13%] (constant error for Distance)

Table 12: Distance the Earth is from the Sun at the times of the Sun's Zenith

Date	Time 1 ± 60s	Distance (au) ± 0.0013
02-Apr-03	13:10	0.999578
03-Apr-03	13:09	0.999858
04-Apr-03	13:09	1.000139
05-Apr-03	13:09	1.000419
06-Apr-03	13:09	1.000700
07-Apr-03	13:08	1.000980
08-Apr-03	13:08	1.001260
09-Apr-03	13:08	1.001541
10-Apr-03	13:08	1.001821
11-Apr-03	13:07	1.002102

5. Distance from the Sun to Jupiter

The Earth-Jupiter ephemeris obtained from NASA gave the distance from the Sun to Jupiter over the relevant days.

Table 13: Data from the Earth -Jupiter Ephemeris[11]

Date (0 TDT)	Distance from Sun to Jupiter (a.u.)
01-Apr-03	5.33096
03-Apr-03	5.33158
05-Apr-03	5.3322
07-Apr-03	5.33282
09-Apr-03	5.33344
11-Apr-03	5.33405
13-Apr-03	5.33467

---

[11] NASA/Goddard Space Flight Center
Geocentric Ephemeris for the Sun; Fred Espenak.
Available from the World Wide Web:<http://sunearth.gsfc.nasa.gov/eclipse/TYPE/TYPE.html>

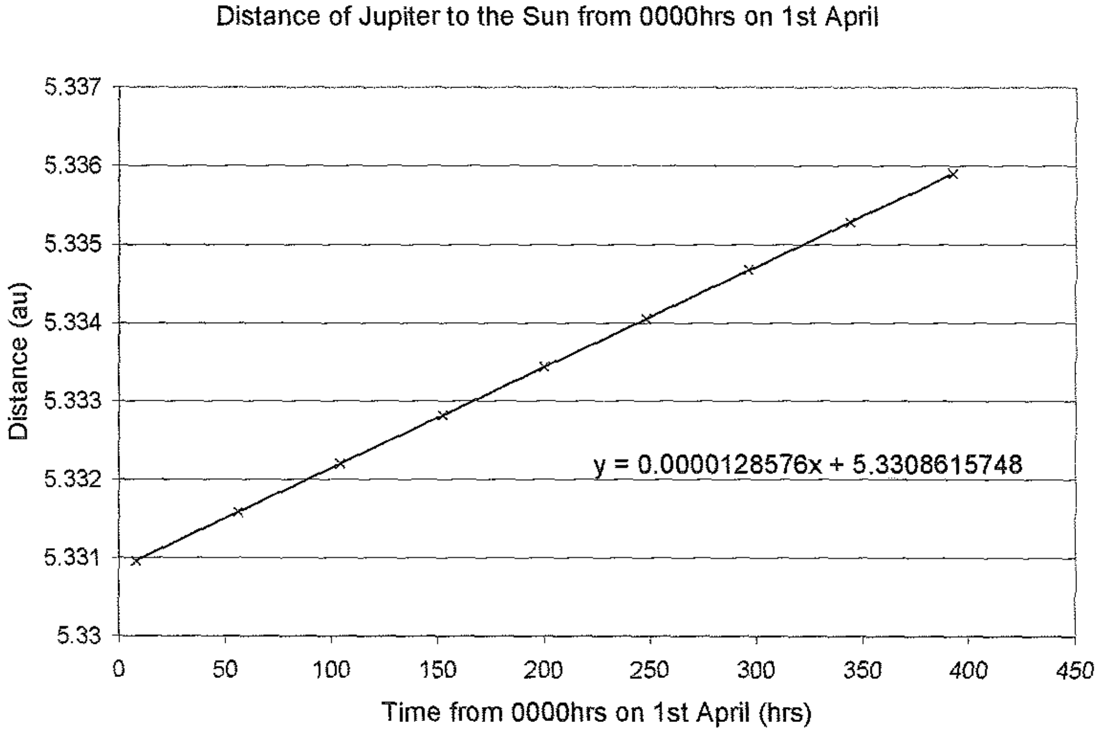
Results: Calculating the distance from the Sun to Jupiter

Table 14: Conversion from TDT to GMT +0800hrs

Time from the 1st April 0000hrs (hrs)	Distance from the Sun to Jupiter (au)
8.017944	5.33096
56.01794	5.33158
104.0179	5.3322
152.0179	5.33282
200.0179	5.33344
248.0179	5.33405
296.0179	5.33467

As explained earlier below Table 11, the error in the data is small but due to the size of other errors that have been taken into account, this small increase in distance is taken into account.

Graph 8: Distance of Jupiter to the Sun



The equation is y = 0.0000128576x + 5.3308615748, where y is the distance of Jupiter from the Sun and x is the time from 0000hrs on the 1st of April.

Sample Calculation 7: Distance Jupiter is from the Sun at the times of Jupiter's zenith

2^{nd} April 2003

Time: 21:09

Time from 1^{st} April 0000hrs: $21\frac{3}{20}hrs + 24hrs \times 1$ day from 1^{st} April $= 45\frac{3}{20}hrs$

[% error = 0.24%] (constant % error for Time)

Distance: $0.0000128576 \times 45\frac{3}{20}hrs + 5.3308615748 = 5.33144\text{au} \pm$ 0.0128

[% error = 0.24%] (constant error for Distance)

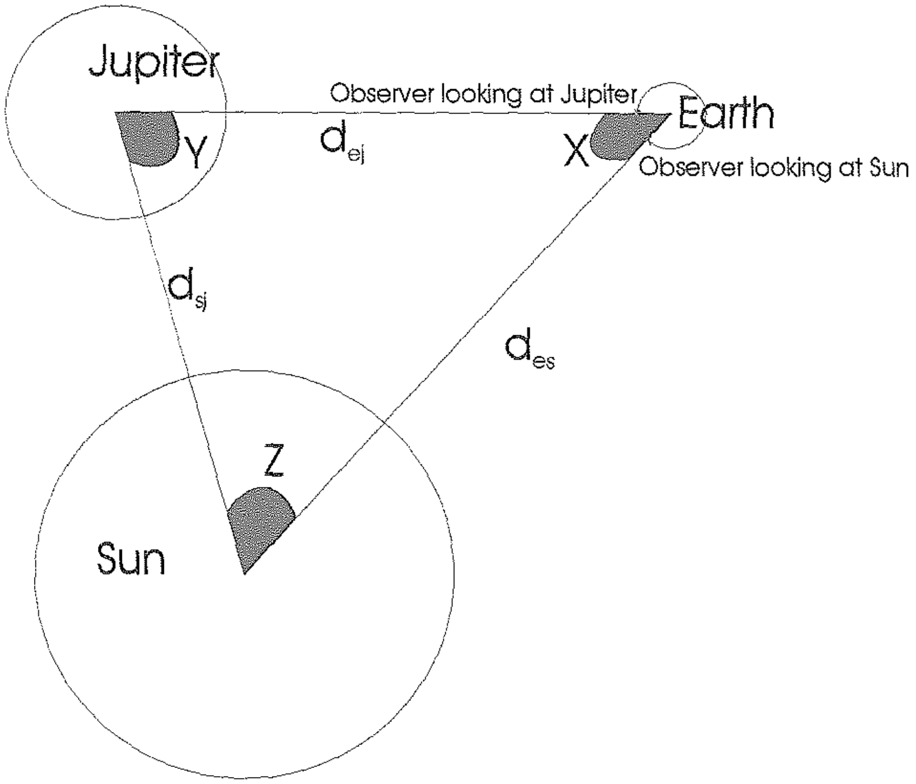
Table 15: Distance Jupiter is from the Sun at the times of Jupiter's Zenith

Date	Time 2 ± 180s	Distance (au) ± 0.0128
02-Apr-03	21:09	5.33144
03-Apr-03	21:05	5.33175
04-Apr-03	21:01	5.33206
05-Apr-03	20:57	5.33237
06-Apr-03	20:53	5.33267
07-Apr-03	20:50	5.33298
08-Apr-03	20:46	5.33329
09-Apr-03	20:42	5.33360
10-Apr-03	20:38	5.33390
11-Apr-03	20:34	5.33421

Distance from Jupiter to Earth

Having obtained the necessary data, we can calculate the distance from Jupiter to Earth using trigonometry.

Diagram 6: Distance from Jupiter to Earth



Known values: d_{es}, d_{sj} and X
Value to be found: d_{ej}

Using the Sine Rule to find Y, $\frac{\sin Y}{d_{es}} = \frac{\sin X}{d_{sj}}$.

So, $\sin Y = \frac{d_{es} \sin X}{d_{sj}}$ and therefore, $Y = \sin^{-1}\left(\frac{d_{es} \sin X}{d_{sj}}\right)$

Hence Z can be found as Z = 180 - X - Y

Finally using the cosine rule, d_{ej} can be found, as $d_{ej} = \sqrt{d_{sj}^2 + d_{es}^2 - 2d_{sj}d_{es}\cos Z}$

Results: Distance from Jupiter to Earth

Sample Calculation 8: Calculating d_{ej}

d_{sj} = 5.33144au ± 0.0128 [% error = 0.24%]
d_{es} = 0.999578au ± 0.0013 [% error = 0.13%]
X = 119.750° ± 1

Calculating Y

$$Y_{\min} = \sin^{-1}\left(\frac{0.999578\text{au} \times \sin(119.750\text{ -}1)^{\circ}}{5.33144\text{au}}\right) = 9.24^{\circ}$$

$$Y = \sin^{-1}\left(\frac{0.999578\text{au} \times \sin(119.750^{\circ})}{5.33144\text{au}}\right) = 9.37^{\circ}$$

$$Y_{\max} = \sin^{-1}\left(\frac{0.999578\text{au} \times \sin(119.750+1)^{\circ}}{5.33144\text{ au}}\right) = 9.50^{\circ}$$

Error of Y = $\frac{9.50-9.24}{2}$ = 0.13

Calculating Z

Z = 180 - 119.75 - 9.37 = 50.88° ± 1.13

Calculating error in cos(Z)

$\cos(50.88\text{-}1.13)=0.65$
$\cos(50.88)=0.63$
$\cos(50.88+1.13)=0.62$

Error of cos(Z) = $\frac{0.65-0.62}{2}$ =0.0153 [% Error = 2.4%]

Calculating d_{ej}

$$d_{ej} = \sqrt{5.33144\text{au}^2 + 0.999578\text{au}^2 - 2 \times 5.33144\text{au} \times 0.999578\text{au} \times \cos(50.88^{\circ})} = 4.7643\text{au}$$

Error = $\frac{0.24\% \times 2 \times 5.331\text{au}^2 + 0.13\% \times 2 \times 0.9996\text{au}^2 +}{(0.13\% + 0.24\% + 2.4\%) \times 2 \times 5.331\text{au} \times 0.9996\text{au} \cos(50.88^{\circ})}$ = 0.03

Table 16: Calculations of d_{ej}

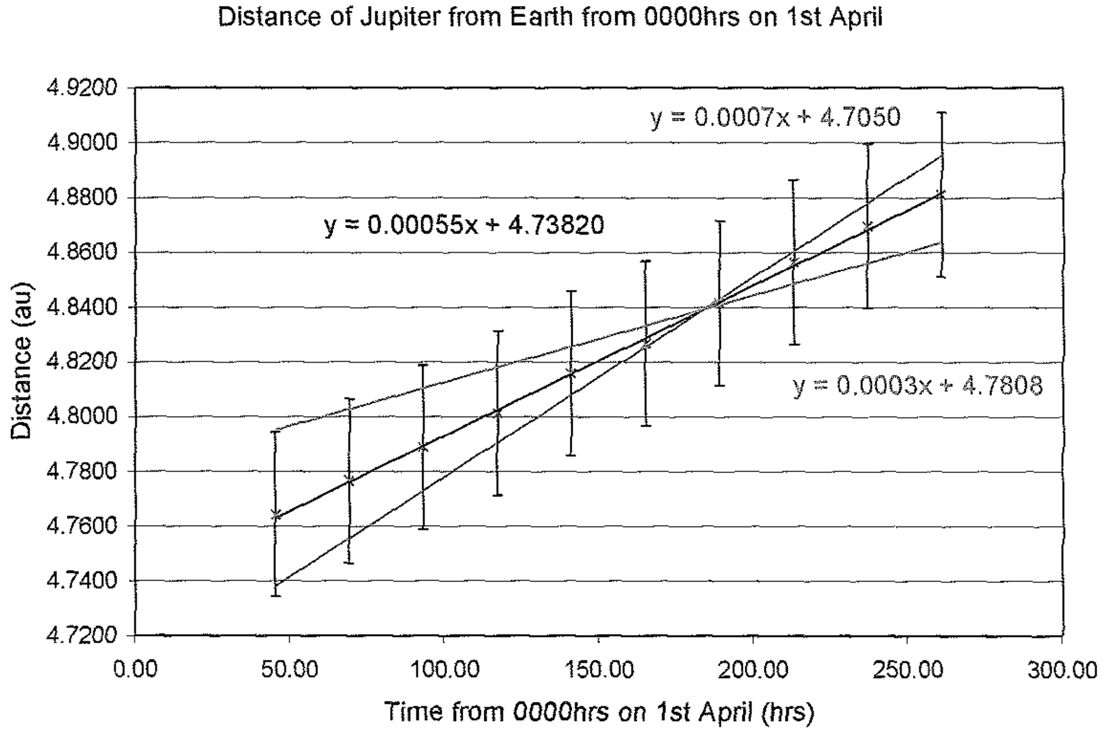
d_{sj} (au) ± 0.0128	d_{es} (au) ± 0.0013	X° ± 1	Y_{min}°	Y°	Y_{max}°	Error of Y	Z°	Error of Z	d_{ej} ± 0.03
5.33144	0.999578	119.750	9.24	9.37	9.50	0.13	50.88	1.13	4.7643
5.33175	0.999858	118.875	9.32	9.45	9.58	0.13	51.67	1.13	4.7765
5.33206	1.000139	118.000	9.41	9.53	9.66	0.12	52.47	1.12	4.7889
5.33237	1.000419	117.125	9.49	9.61	9.73	0.12	53.26	1.12	4.8014
5.33267	1.000700	116.125	9.58	9.70	9.82	0.12	54.18	1.12	4.8158
5.33298	1.000980	115.375	9.65	9.76	9.88	0.12	54.86	1.12	4.8268
5.33329	1.001260	114.375	9.73	9.85	9.96	0.12	55.78	1.11	4.8415
5.33360	1.001541	113.375	9.81	9.93	10.04	0.11	56.70	1.11	4.8564
5.33390	1.001821	112.500	9.88	9.99	10.10	0.11	57.51	1.11	4.8696
5.33421	1.002102	111.750	9.94	10.05	10.16	0.11	58.20	1.11	4.8810

Table 17: The time of Jupiter's Zenith and the distance Jupiter is from the Earth

Date	Time 2 ± 180s	d_{ej} ± 0.03
02-Apr-03	21:09	4.7643
03-Apr-03	21:05	4.7765
04-Apr-03	21:01	4.7889
05-Apr-03	20:57	4.8014
06-Apr-03	20:53	4.8158
07-Apr-03	20:50	4.8268
08-Apr-03	20:46	4.8415
09-Apr-03	20:42	4.8564
10-Apr-03	20:38	4.8696
11-Apr-03	20:34	4.8810

This time is used because it is when a straight line goes from the center of Jupiter to the center of Earth through the point of the observer. A graph, time against distance, is plotted and equations found.

Graph 9: Distance of the Jupiter from the Earth



Three possible equations were found showing the relationship between time and distance from Earth to Jupiter. Again, y is the distance from Earth to Jupiter in astronomical units and x is the time in hours from 0000hrs on 1st April.

Distance Jupiter is from Earth at the time of Io's maximum displacement

In Table 1, the maximum displacement of Io is highlighted in red. The time the picture was taken from 1st April 0000hrs is 49.0906 hrs.
This is used to calculate the distance from Earth Jupiter is at that time.
Min Distance: $0.0007 \times 49.0906 + 4.7050 = 4.7394au$
Distance: $0.00055 \times 49.0906 + 4.73820 = 4.7652au$
Max Distance: $0.0003 \times 49.0906 + 4.7808 = 4.7955au$
Error = $\frac{4.7955 - 4.7394}{2}$ = 0.028 [% error = 0.59%]

According to NASA, 1 au = 149,597,870 km[12], hence the distance 4.7652 au ± 0.028 is converted into 7.13×10^8 km ± 4.2×10^6.

Conclusion

The distance of Jupiter from Earth is 7.13×10^8 km± 4.2×10^6. According to the NASA Jupiter-Earth Ephemeris, the distance of Jupiter from Earth is 7.22×10^8 km meaning that the calculated value is close to the real value.

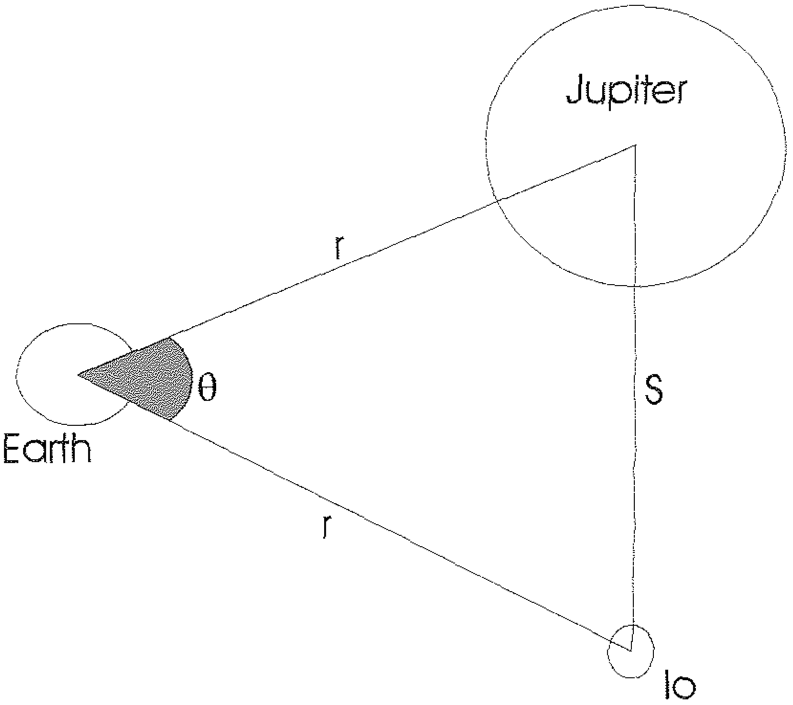
---

[12] NASA - Imagine the Universe
Ask a High Energy Astronomer; David Palmer.
Available from the World Wide Web:
<http://imagine.gsfc.nasa.gov/docs/ask_astro/answers/980122b.html>

Section D: Radius of Io and Europa's orbit

The radius of the orbit was calculated using $S = \theta r$, where S is the radius of the orbit, θ is the angular radius and r is the distance from Earth to Jupiter.
This formula was used because of the small angle involved, as can be seen in the below diagram 7, using Io as the example.

Diagram 7: Calculation of the Io's orbit



The small angle θ means that an approximation is made that the distance from the center of Earth to the center of Io is approximately equal to the distance from the center of Earth to the center of Jupiter. Also, the small angle means that the side of a circle is approximately equal to a straight line, so we can use $S = \theta r$.

Radius of Io's orbit

The angular radius of Io's orbit: 6.56×10^{-4} rad
[% error =9.75%]
Distance from Earth to Jupiter at the time of maximum displacement: 7.13×10^{8} km $\pm 4.2 \times 10^{6}$
[% error = 0.6%]

Sample Calculation 9: Radius of Io's orbit

$6.56 \times 10^{-4}\ \text{rad} \times 7.13 \times 10^{8}\ \text{km} = 467728\ \text{km}$ [% error = 10.34%]

Conclusion

NASA states that Io's orbit is 421,600 km[13], which is fairly close to *the figure* calculated 467728 km.

Radius of Europa's Orbit

The same calculation applies to Europa.
The angular radius of Europa's orbit: 0.0012 rad
[% *error* =7.15%]
Distance from Earth to Jupiter at the time of maximum displacement: 7.13×10^{8} km ± 4.2×10^{6}
[% error = 0.6%]

Sample Calculation 10: Radius of Europa's orbit

$0.0012\ \text{rad} \times 7.13 \times 10^{8}\ \text{km} = 855600\ \text{km}$ [% error = 7.7%]

Conclusion

NASA states that Europa's orbit is 670,900 km[14], which is considerably smaller than the figure calculated which is 855600 km. This is possibly because the radius of Europa's orbit is an approximation as photos were not taken at the estimated time of maximum displacement.

---

[13] NASA - Gallileo: Journey to Jupiter
Io Fact Sheet; Shannon McConnell.
Available from the World Wide Web:<http://www.jpl.nasa.gov/galileo/io/fact.html>

[14] NASA - Gallileo: Journey to Jupiter
Europa Fact Sheet; Shannon McConnell.
Available from the World Wide Web:<http://www.jpl.nasa.gov/galileo/europa/>

Section E: Mass of Jupiter

Mass of Jupiter using the radius of Io's orbit

Now, Kepler's Law, $M_J = \frac{4\pi^2 r_S{}^3}{GT_S{}^2}$, is used to calculate the mass of Jupiter.

$T_S = 42hrs = 151200\text{s}$

$r_S = 467728\text{km}=467728000\text{m}$

$M_J = \frac{4\pi^2 \times 467728000\text{m}^3}{G \times 151200\text{s}^2} = 2.65 \times 10^{27} kg$

% error = 10.34% x 3 ≈ 31%
Error = 8.2 x 10^{26}

Mass of Jupiter using the radius of Europa's orbit

$T_S = 85.3hrs = 307080\text{s}$

$r_S = 855600\text{km}=855600000\text{m}$

$M_J = \frac{4\pi^2 \times 855600000\text{m}^3}{G \times 307080\text{s}^2} = 3.93 \times 10^{27} kg$

% error = 7.74% x 3 ≈ 23%
Error = 9.1 x 10^{26}

Conclusion

The mass of Jupiter found is 2.65 x 10^{27}kg ± 8.2 x 10^{26} and 3.93 x 10^{27}kg ± 9.1 x 10^{26}, using data from Io's and Europa's orbit respectively, while NASA states that the mass of Jupiter is 1.9 x 10^{27} kg.[15]

The figure from Io is fairly accurate as the mass according to NASA lies within the range of possible values due to the uncertainties. However the mass from Europa is about twice the size and may be due to the fact that Europa was not the focus of the investigation and inadequate information was obtained about Europa.

---

[15] Students for the Exploration and Development of Space (SEDS). **The Nine Planets: A *Multimedia Tour of the Solar System***; Bill Arnett. Available from World Wide Web:<http://seds.lpl.arizona.edu/nineplanets/nineplanets/jupiter.html>

Evaluation

There were several difficulties in conducting this investigation and one of which was the time frame in which Jupiter could be seen. Because of the location of the telescope, Jupiter could only be seen between 10pm and 2am. Furthermore, because the investigation was limited to a short period of time, only Io and Europa could be focused upon because the other satellites, Ganymede and Callisto, have orbits which are larger and the times of maximum displacement was not between 10pm and 2am. It was difficult to collect enough data for Ganymede and Callisto due to the length of their orbits.
If this investigation was done again, the investigation would take place over a longer period of time. Also all four of the main satellites would be taken into account, so to give more accurate of data. For each of the four satellites, data has to be obtained at the maximum displacement otherwise the sine curve found is inaccurate as found with Europa.

The site the telescope was located at had the disadvantage which was Jupiter could only be seen between 10pm and 2am. If the telescope was moved to a more open location, Jupiter could be seen from 7.30 pm to 2am, almost doubling the time. However, moving to a more open location loses the advantage of the present location as it would also be more open to the elements and the telescope may get spoilt being exposed to rain. But more data could be obtained if the telescope was at a different site.

If the telescope had a larger magnification, the small satellites would be seen more easily as they would be larger and hence reduce the percentage error. However the downside to larger magnification is the same amount of light gives more detail, leaving the image darker. To correct this problem, a larger aperture is needed to increase the amount of light let in.

The camera too could have a longer zoom, so as to see the small satellites more clearly. However again this would result in the same amount of light giving more detail and hence more light needs to be let into the camera image.

The cardboard ring that held the camera to the telescope was suitable for its purpose but it was not firm. The camera did occasionally slide slightly back and forth as the cardboard ring moved slightly. To improve this, a better ring could be made, something stronger and better fitting so to keep the camera firmly attached to the telescope.

References

1. Students for the Exploration and Development of Space (SEDS).
The Nine Planets: A Multimedia Tour of the Solar System; Bill Arnett.
Available from World Wide Web:
<http://seds.lpl.arizona.edu/nineplanets/nineplanets/jupiter.html>

2. Giancoli, Physics, Fifth Edition, Prentice Hall International (UK) Limited, London, 1980 Pg 133

3. Department of Earth Sciences, Simon Fraser University.
Global Geophysics; Dr. A. C. Calvert. Available from the World Wide Web:
<http://www.sfu.ca/earth-sciences/courses/317/Chap1/1-Introduction.htm>

4. NASA - Gallileo: Journey to Jupiter
Io Fact Sheet; Shannon McConnell.
Available from the World Wide Web:
<http://www.jpl.nasa.gov/galileo/io/fact.html>

5. NASA - Gallileo: Journey to Jupiter
Europa Fact Sheet; Shannon McConnell.
Available from the World Wide Web:
<http://www.jpl.nasa.gov/galileo/europa/>

6. Redshift 3; United Soft Media; November 1998

7. NASA/Goddard Space Flight Center
Geocentric Ephemeris for the Sun; Fred Espenak.
Available from the World Wide Web:
<http://sunearth.gsfc.nasa.gov/eclipse/TYPE/TYPE.html>

8. Delta T: Approximate Algorithms for Historical Periods; Robert H. van Gent.
Available from the World Wide Web:
<http://www.phys.uu.nl/~vgent/astro/deltatime.htm>

9. NASA/Goddard Space Flight Center
Delta T and Universal Time; Fred Espenak.
Available from the World Wide Web:
<http://sunearth.gsfc.nasa.gov/eclipse/SEhelp/deltaT.html>

10. NASA - Imagine the Universe
Ask a High Energy Astronomer; David Palmer.
Available from the World Wide Web:
<http://imagine.gsfc.nasa.gov/docs/ask_astro/answers/980122b.html>

Bibliography

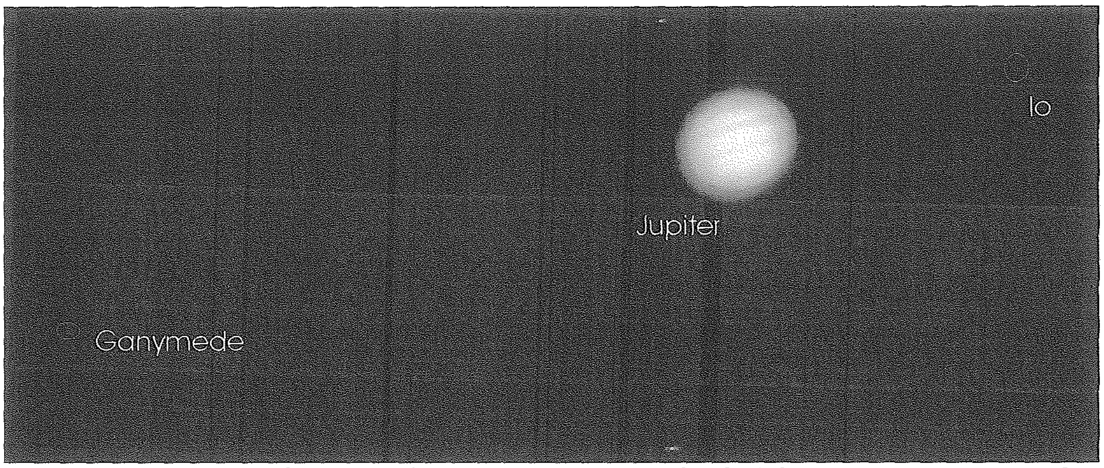
1. Students for the Exploration and Development of Space (SEDS).
The Nine Planets: A Multimedia Tour of the Solar System; Bill Arnett.
Available from World Wide Web:
<http://seds.lpl.arizona.edu/nineplanets/nineplanets/jupiter.html>
2. Giancoli, Physics, Fifth Edition, Prentice Hall International (UK) Limited, London, 1980
3. Department of Earth Sciences, Simon Fraser University.
Global Geophysics; Dr. A. C. Calvert. Available from the World Wide Web:
<http://www.sfu.ca/earth-sciences/courses/317/Chap1/1-Introduction.htm>
4. NASA - Gallileo: Journey to Jupiter
Io Fact Sheet; Shannon McConnell.
Available from the World Wide Web:
<http://www.jpl.nasa.gov/galileo/io/fact.html>
5. NASA - Gallileo: Journey to Jupiter
Europa Fact Sheet; Shannon McConnell.
Available from the World Wide Web:
<http://www.jpl.nasa.gov/galileo/europa/>
6. Redshift 3; United Soft Media; November 1998
7. NASA/Goddard Space Flight Center
Geocentric Ephemeris for the Sun; Fred Espenak.
Available from the World Wide Web:
<http://sunearth.gsfc.nasa.gov/eclipse/TYPE/TYPE.html>
8. Delta T: Approximate Algorithms for Historical Periods; Robert H. van Gent. Available from the World Wide Web:
<http://www.phys.uu.nl/~vgent/astro/deltatime.htm>
9. NASA/Goddard Space Flight Center
Delta T and Universal Time; Fred Espenak.
Available from the World Wide Web:
<http://sunearth.gsfc.nasa.gov/eclipse/SEhelp/deltaT.html>
10. NASA - Imagine the Universe
Ask a High Energy Astronomer; David Palmer.
Available from the World Wide Web:
<http://imagine.gsfc.nasa.gov/docs/ask_astro/answers/980122b.html>
11. G. R. Fuller, Optical Systems, Heinemann Educational Australia PTY LTD, Victoria, 1970
12. Dr. Richard Tresch Fienberg, Sky & Telescope, Sky Publishing Corp, Cambridge, Massachusetts, 2003

NB: *Photo on front cover was taken by the Hubble Space Telescope on February* 13th 1995

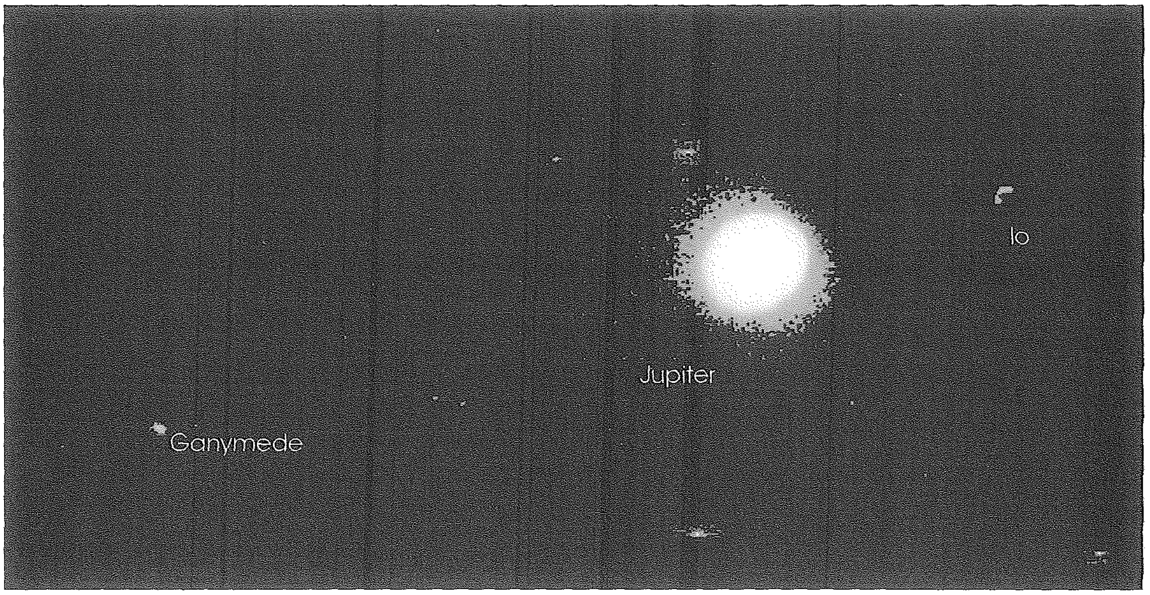
Appendix 1: Sample of an underexposed photo

Photo taken 2nd April 22:58

As it can be seen, the satellites are faint and only with the ability to manipulate the image using tools provided by Microsoft Photo Editor, are they visible.



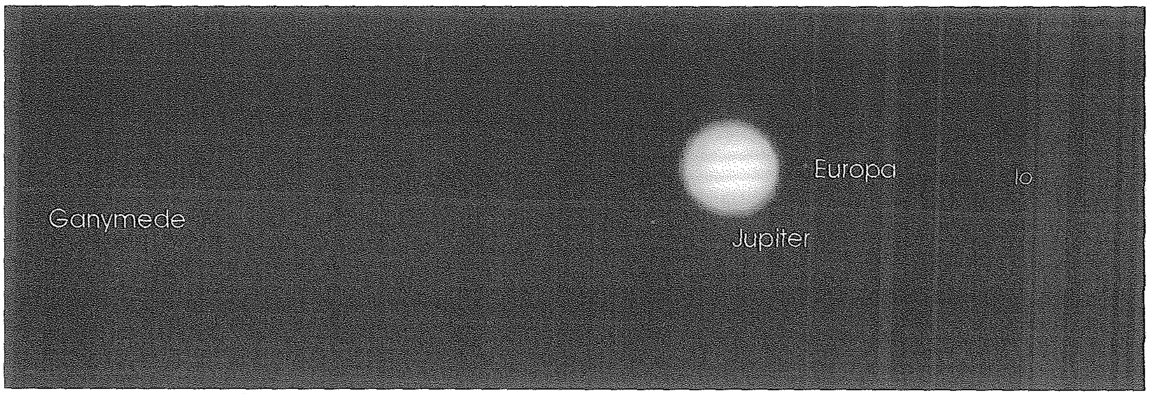
Below is the same photo as the one above but the brightness, contrast and gamma features have been manipulated in order to be able to see Io and Ganymede clearly. The manipulation of the photos in this manner does not add or delete any pixels and by careful comparison of the two photos, it is possible to figure out the pixel coordinates.



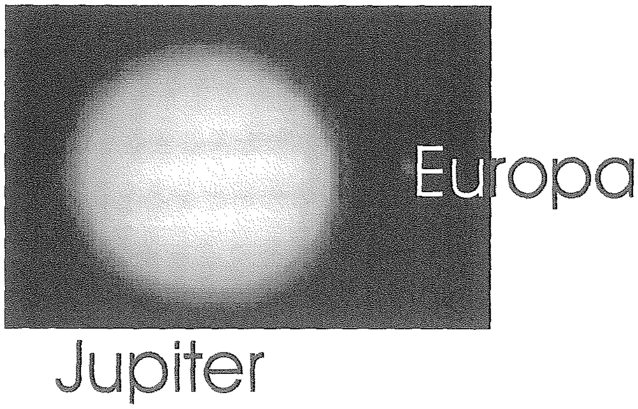
Appendix 2: Sample of a photo taken at Normal Settings

Photo taken 3rd April 00:02

The satellites can be made out as faint grey specks, but with the software to magnify the picture, it is possible to see the satellites clearly enough to obtain pixel coordinates.



Having magnified the photo, Europa can be seen more clearly than in the larger photo. This facilitates finding the pixel coordinates of the edges of the satellite.



Appendix 3: Sample of over exposed photo

Photo taken 3rd April 00:01

The satellites are clearly visible in the over exposed photos. However, it can be seen that the edges of Jupiter are blurred.