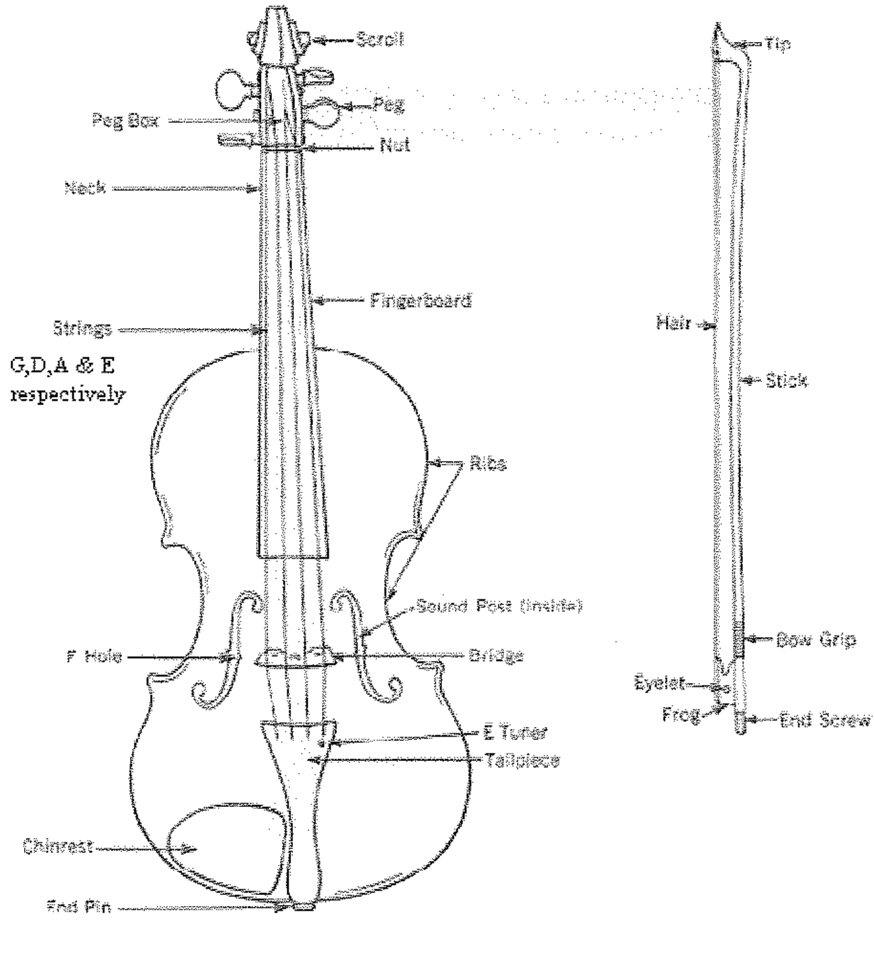
Physics and the Violin



Contents

Abstract

The subject of my essay, "**How physics has been used to improve the quality of a violin**", is a very broad subject. Therefore my essay was narrowed down, including mainly ideas concerning the construction of the violin, to allow for detail and discussion

- To begin my essay I inform the reader of the history of the violin. This is to give the reader an understanding of how the violin has progressed to its current status.

- The next section describes the violin as it is today. This serves two purposes. Firstly, it tells the reader what it is that I am actually writing about and secondly, it allows the reader to make comparisons with the history section of the essay. Hopefully, the reader starts to appreciate the long and windy road that the violin has taken over the centuries.

- In this section of the essay I talk about Mersenne's Laws. These are at the basis of all string instruments. Without understanding of these laws and how they apply to the violin, the understanding of the whole topic of my essay wouldn't be possible.

- The next two sections of my essay deal with physical ideas specifically related to the violin. How the sound is emitted from it and the idea of resonant frequencies. The sound idea provides links to other areas of the broader subject such as physical techniques the player can employ and the resonant frequency section allows for fairly in depth physics to be included.

- Finally, the final section, the conclusion, gives me an opportunity to answer the essay question and also suggest to the reader how the subject of my essay can be extended to further answer the essay question.

Word Count - 3980

Describe the violin and discuss how physics has been used to improve the quality of sound emitted.

In this essay I will try to explain how some aspects of physics have been used to maximise the quality of sound produced from the violin. I will do this with the help of some personal practical evidence I devised, to reinforce or serve as explanatory aids for me to base my writing around.

History and Background

The violin is thought to have first appeared sometime around the 15th century. However, its precursors can be traced to as far back as 5000BC. In Asia, predominantly India, the earliest relative of the violin is said to be the Ravanastron. This instrument was brought to Europe by Muslim merchants and artists. Then, a gradual ascendancy took place right up until the 18th century. Over the years the primitive instruments became more and more complicated, evolving fingerboards and numerous more strings leading to a vast range of sound that is perhaps the reason for its success. Shortly after the 1500's, the family split into three directions. Firstly, there were the viola da gamba, or viols played between the knees. Then there were the lire da braccio, similar instruments played with a bow and the viola da braccio, instruments played against the shoulder. The violin originates from the viola da braccio group. From this group the violin evolved the four strings tuned in fifths and the f-shaped sound holes to give birth to the most versatile and manoeuvrable instrument made to this date, in my opinion. In the 17th and 18th centuries the violin seed was planted across Europe in countries such as, Germany, France, Czechoslovakia, Italy and England. Chamber music, performed and written by Mozart and Haydn being a widely accepted reason for this. However, the flower blossomed most beautifully in Italy, which became the violin and arts capital of the world for a great many years. The final major development of the violin came during the 17th century with a man named Vivaldi. The violin became a recognised soloist instrument, often accompanied by whole orchestras and pianos. Vivaldi made this transformation in order to show off his technical abilities on the violin and consequently, dazzle and inspire future generations. Unfortunately from here on, the violin began its slow decline to modern times. This may be due to the decline of Baroque music as the violin was very representative of this style. Also, new genres of music were developed for example, jazz and rock. However, as music advanced during the renaissance, the violin became an irreplaceable part of the orchestra and has consequently survived to the present date. (1), (3) & (4)

Construction of the violin

The violin is an extremely complicated instrument. It has numerous different gadgets, which all interrelate and work together to acheive a projection of sound. For example, the bow produces vibrations in the violin string due to tiny hooks on the bow hairs. These vibrations are then transferred through the bridge and down the sound post situated within the body of the violin. These vibrations are then relayed onto the body of the violin to produce tiny pressure changes in the surrounding air, which travel through the air to people's ears as sound, or music. This example shows the intimate relationship between all the parts of the violin, of which, more will be explained later on in this essay. Other important parts of the violin it is useful to know about are, the fingerboard, a plateau on which the violinist pushes down the string to produce a particular note, the peg box, a series of pegs which when turned either tighten or losen the string, used to tune the strings to the correct pitch and the f-holes, two holes on the body of the violin with *f* shapes designed to allow the flow of air in and out of the sound box or body of the violin. For a clearer idea as to the whole picture of the violin it is possible to refer to the diagram on the title page of this essay. It is also interesting to know that due to lack of development, violins made by old famous violin makers, these being generally accepted as Stradivari and Guarneri, will now fetch millions of pounds.

Mersenne's Laws

The first person believed to have considered the physics of music or instruments was Pythagoras. He proved mathematically, the existence of modes of vibration, and found mathematical links between them in musical instruments (8). Unfortunately, as with most scientific subjects, very little progress was made in the field of design and construction of stringed instruments after the golden years of Pythagoras. However, following this void in progress, Galileo (1564 – 1642) was the first to find the mathematical relationship between the frequency, length, tension, diameter and density of the vibrating string. However, the name we associate with these findings, Mersenne, did the experiments independently and published his findings before Galileo even though he arrived at his conclusions afterwards. These laws, at the basis of all mechanics of the violin, have been the most influential in improving the quality of the violin.

Mersenne's laws, for all string instruments are as follows (1): -

- **The frequency of a string is inversely proportional to the length.**
- **The frequency of a string is directly proportional to the square root of the tension.**
- **The frequency of a string is inversely proportional to the diameter for wires of the same material.**
- **The frequency of a string is inversely proportional to the mass per unit length.**

All these laws can be explained by looking at the violin itself.

Violinists exploit the first law through use of the fingerboard. By this I mean that when a player puts a finger down on the fingerboard, he/she in effect shortens the length of vibrating string. As the frequency is inversely proportional to the length of string, the frequency of the note produced becomes higher. Therefore, it follows that the further towards the bridge your fingers push down on the string, the higher the pitch, of the note produced.

To prove this law I performed a small experiment. This was done on a sonometer. This is a hollow box with a single string running above it passed over one fixed bridge and one moveable bridge. To keep the string taught a mass of 500g was hung on the end of the string while the other end was attached to the fixed bridge. By moving the moveable bridge, I could vary the length of vibrating string and the frequency was measured on a cathode ray oscilloscope. The results and consequent graph produced proves the inversely proportional relationship between the frequency and length of string.

To extend this point I decided to perform another small experiment to calculate the error with which an experienced violinist can detect a change in frequency. As a result I can give you some perspective into the accuracy needed to play in tune on the violin. The experiment was performed on a guitar as the notes resonate more and therefore last longer than on the violin allowing more time to perform the experiment. To measure the frequency of the note produced, an electronic tuner was used. For the experiment, I plucked the starting note and pushed down on the string behind the nut to gradually increase the frequency and pitch of the note. When my sister said she could recognise the difference in note, I recorded the difference in Hertz the note was from the frequency of the starting note. This turned out to be a difference of 1Hz. To work out the difference in length a string must be to produce a difference of 1 Hz an equation was derived. This equation is as follows: -

$$\frac{\textbf{Distance from bridge to nut}}{\textbf{Distance from finger to bridge}} \times \textbf{frequency of open string} = \textbf{frequency of note played}$$

This equation was derived using Mersenne's first law (frequency is inversely proportional to length). In other words as the distance from the finger pressing down and the bridge doubles, the frequency will half, as shown in the equation. Using this equation, it is possible to work out the frequency of a known note, then increase the frequency of this note by one and work out the difference in distance between finger and bridge. For my sister, the distance her finger must be out of place to hear a wrong note is 0.012cm (calculation can be found on appendix 7), or almost a tenth of a millimetre. The minute magnitude of this value is such that the accuracy required from a violin player when playing any notes is phenomenal. Not only does this experiment shed light onto the difficulty of playing the violin but the physical ability of the great violinists, such as Paganinni, renowned for the speed that he could play at. These violinists, probably with much keener hearing ability than my sister, might be able to play up to 10 notes per second perfectly in tune! What's more, from personal experience I know that the margin for error is even smaller the higher up the violin you get.

In evaluation of this experiment, the main problem with it is that only one person actually performed the experiment. There is most likely a broad range of hearing abilities across the world population and my experiment is not representative of this. Also, the method was quite crude and the electronic tuner used wasn't accurate to more than ± 1 Hertz. To improve this experiment I could have used a cathode ray oscilloscope to measure the change in frequencies and used a larger sample of people to perform the experiment on. An interesting extension to this experiment could be to investigate how this margin for error changes at different points along the string. For example, does the margin for error decrease as you move closer to the bridge?

Proof of the second law, that the frequency of a note is directly proportional to the square root of the tension is that the lower frequency strings on the violin are much slacker than the higher frequency strings. Further evidence for this comes from the technique of tuning these strings. For example, if the string was too low in pitch then you would wind a peg at the top end of the violin, next to the scroll, which acts as a kind of winch. It subsequently tightens the string increasing its tension and therefore increasing its pitch slightly therefore tuning the string to its correct frequency.

Similarly to the first law, I performed an experiment as a proof of the second law. The same apparatus was used, however, the moveable bridge was kept in a fixed position and the masses were added uniformly to the string being investigated. Again, the results and graph obey Mersenne's second law that the frequency is directly proportional to the square root of the tension.

The third and fourth laws, frequency is inversely proportional to both the diameter and the mass per unit length, can be viewed as one. This is because by increasing the diameter of a string, you are, in effect, also increasing the mass per unit length. These laws can be shown in the everyday use of stringed instruments in the way that as the string gets lower in pitch, if the length is constant, the string gets thicker and thicker therefore increasing its diameter and mass per unit length.

For these laws, as for the others, experiments were performed on the sonometer. However, for these variables there is an inverse square relationship involved. This can be proved by the following equation: -

$$\text{Mass per unit length } (\mu) = \frac{\text{Mass}}{\text{Length(L)}}$$

$$= \frac{\text{Density}(\rho) \times \text{Volume}}{L}$$

$$= \frac{\rho \times \text{Area} \times \cancel{L}}{\cancel{L}}$$

$$= \rho \times \text{Area}$$

$$= \rho \times \pi r^2$$

$$= \rho \times \pi \cdot \left(\frac{\text{Diamete.}}{2}\right)^2$$

$$= \rho \quad \times \quad \frac{\pi \cdot D^2}{4}$$

Therefore: -

$$\mu \quad \alpha \quad D^2$$

In other words, if the diameter is doubled then the mass per unit length increases by a factor of 4 or 2^2. Therefore, the frequency must be inversely proportional to one variable (diameter) and inversely proportional to the square of the other variable (mass per unit length). To view all the results and graphs on Mersenne's Laws please refer to appendices 1, 2, 3, 4, 5, 6, and 7.

Problems encountered in these Mersenne's Laws experiments were mainly due to lack of equipment. Especially while investigating the mass per unit length and diameter of the string, as only four strings were available (the four violin strings). This limited the number of results I was able to take. To improve this experiment I would need to acquire more strings of different diameter and mass per unit length. To extend this investigation I could look the ratios between the tensions, mass per unit lengths, diameters and lengths of strings on different violins and perhaps compare higher and lower quality violins and comment on any differences.

Sound Distribution around a violin

One other area into which physics has shed insight is the distribution of sound around the violin. Ideally, the violin needs to project sound outwards and upwards towards the audience. This means that the audience hears a louder more pure sound. If the violin didn't do this then the sound could in effect, bounce of more walls and other surfaces and interfere with itself so that the audience hears a sound of less quality.

I decided that in order to explain this properly I could draw a diagram. I invented an experiment to produce a kind of contour map showing where the sound is projected most from the violin. This was done by firstly, fixing the violin in a certain position. A microphone was placed near to the bridge of the violin and a second microphone measured the sound in arbitrary units in 10cm intervals in all perpendicular directions from the bridge. The two amplitudes were displayed on a cathode ray oscilloscope. From these results I can draw two diagrams. One for a vertical cross section of the violin and one for a horizontal cross section of the violin. The purpose of the microphone in a fixed position was to ensure that the loudness of the sound generated from the violin was kept constant. Both microphones were connected to a Cathode Ray Oscilloscope, from which I could record the volume of the sound picked up from both microphones.

The main problem with this experiment is that it was very hard to look at the two readings on the cathode ray oscilloscope simultaneously. Therefore, it was equally difficult to ensure that each pluck of the string was perfectly equal. Also, the microphones used were not identical. To improve this experiment I believe it should be performed with two people, one ensuring that each pluck is of the same amplitude and one recording the sound levels at different points around the violin. Also, a mechanical plucker could be used to make each pluck more equal to the next. For example, a flexible piece of wire that gives under a certain pressure. Finally, the two microphones should be equal in order to give accurate results.

From the diagrams I can see that firstly, as you get further away from the violin in all directions the loudness decreases and that the loudest point around the violin is just above the bridge, in line with both the f-holes. Also, from the horizontal cross section I can see that more sound is emitted from the sides of the violin, where the f-holes are placed, than from the ends, where the scroll and chin rest are situated. Finally, from the vertical cross section I can see that more sound is emitted from the top of the violin than from the bottom of the violin. To see all results and diagrams you can refer to appendices 8, 9, 10 and 11.

By looking at the violin itself I can see certain traits that explain this knowledge. For example, the f–holes are situated on the sides of the violin, which could allow more sound to escape. Also, at the ends of the violin there is a lot of wood and other material in line with the sound centre, the bridge, compared to the sides of the violin, which has relatively little. This wood and general material could interfere with the projection of sound. Similarly, there is more wood in the way of sound going down than sound going upwards. Therefore, the sound is louder above the violin than below. These findings have also led to certain postures being proposed to allow the maximum quality of sound to be projected. By this I mean, it is generally deemed necessary to hold the violin at almost 90 degrees from the direction your body is pointing. Also, to tilt your violin at about 20 to 30 degrees from the horizontal, towards the audience. This way it is possible to stand with your body facing the audience and also the f-holes in line with the direction of the audience. Consequently, the loudest areas around the violin, above it and by the sides of it, are mostly pointing towards the audience so that they receive the loudest and most undisturbed sound.

Resonant Frequencies

The final area of the physics of the violin that is concerned with the quality of sound produced is this idea of the violin's fundamental resonant frequency. In a violin, the sound generator, the string, creates vibrations that transmit down the bridge into the body or sound box of the violin and then in turn to the air in the violin and then the air outside the violin. This is done by a series of pressure changes inside the sound box created by the tiny vibrations in the wood of the sound box. Basically, the sound box contracts slightly with each vibration causing an increase in pressure inside the violin. Air then rushes out of the box through the f-holes due to the pressure gradient with the air outside the box at a lower pressure. Periodically, the sound box then expands a little creating a lower pressure than atmospheric pressure inside of it. Air then rushes back into the sound box due to a similar but reversed pressure gradient as before. As this happens at an extremely fast frequency these tiny pressure changes are transmitted through the air outside the violin into people's ears as waves of sound. These pressure changes are called compressions (high pressure) and rarefactions (low pressure). Resonance occurs when these pressure changes constructively superimpose upon each other inside the sound box. In other words, the frequency is such that the compressions and rarefactions of the air inside the box line up with each other and simply add to each other to create bigger compressions and rarefactions. This happens at many different frequencies, or modes, in physics terminology. These can be investigated through optical holography, the latest in sound experimentation (2). The fundamental resonant frequency is the lowest most frequency at which resonance occurs, it is always the most dominant or noticeable frequency.

In 1984 Lord Raleigh devised an equation to calculate the fundamental resonant frequency of a violin from its volume inside the violin and the area of each f-hole: -

$$f = \frac{0.27 \cdot c \cdot A^{1/4}}{V^{1/2}} \qquad (1)$$

- **f – resonant frequency (Hz)**
- **c – speed of sound (ms^{-1}) ($330ms^{-1}$)**
- **A – area of f-holes (m^2)**
- **V – volume of sound box (m^3)**

For a final experiment, I decided to perform an experiment on Raleigh's equation and deduce the speed of sound graphically. The resonant frequency can be identified by a sharp increase in loudness as the frequency increases over it. The resonant frequency was therefore recorded, using a cathode ray oscilloscope whilst the area of the f-holes was decreased. This was done by shading the area of the f-hole onto a piece of paper, cutting out the shape, drawing around this shape onto a piece of graph paper and then counting the squares inside the outline. As the frequency and area of f-holes were chosen as x and y variables, the gradient then equalled, $\frac{0.27 \cdot c}{V^{1/2}}$.

The volume of my sound box is $0.00185m^3$, calculated by filling the sound box with rice and then pouring it into a volume jug, therefore it is possible to calculate the speed of sound from the gradient. The results, graph and calculations can be found in the appendices. From the gradient, the speed of sound was worked out to be $(309.4 \pm 31.2)ms^{-1}$ The true value for the speed of sound is $330ms^{-1}$ (1), which is within the error of my calculated value, taking into account the inaccuracies of my experiment. All results, graphs and calculations relating to resonant frequencies and Raleigh's equation can be found in appendices 12, 13, 14 and 15.

There are many problems with this more complicated experiment. Firstly, due to the shape of the violin, it was nigh impossible to completely fill it with rice rendering my value for the volume of the violin an underestimate. Secondly, the sizes of the f-holes were changed by placing paper, not wood of the same type as the violin, over the unwanted areas. The paper being more flexible would interfere with the modes of vibration inside the violin and consequently the resonant frequencies. Finally, the resonant frequency was identified by ear introducing more error into the experiment. To improve this experiment I could spend more time in calculating the volume of the violin, ensuring it is as full with rice as possible. Also, I could use violins of different sizes and resonant frequencies instead of tampering with one violin. To extend this investigation I could investigate the differences of resonant frequencies between higher and lower quality violins.

From these experiments, violin makers found that they could make the violin so that its resonant frequency is as close to the ideal value as possible, 285Hz (1) or just above an open d-string on the violin (7), and therefore ensure the violin is of the highest possible quality. They could do this by altering the size of the f-holes and the volume of the sound box. However, later research showed that different types of woods and varnishes also affected the position of the resonant frequency. This is thought to be related to the densities and therefore the rigidity of the sound box walls affecting the modes of vibration of the air.

Conclusion

In conclusion to this essay, I believe that I have shown that physics has played an essential part in the evolution of the violin's quality of sound to the present day. Almost all advances in violin making have stemmed from physical ideas and theories. However, the content of this essay has only touched upon the vastness the violin's relation to physics. By this I mean there are numerous other areas of the violin that originate from scientific ideas. For example, the player's personal technique, the modes of vibration of the body (2) of the violin and the relationship between pressure and closeness to the bridge of the bow (6). Also, it should be mentioned that all experiments performed in relation to this essay do not include repeat readings of any sort. This was due to time constraints whilst carrying out the experiments. Another interesting essay would be to discuss the empirical ideas behind the electronic violin, the most recent style of violin invented. However, despite the fact that science is very fundamental to the violin, the violin was built to create music. Science can explain the creation of sound, but finds it very difficult to explain music, which comes from the soul or emotion of the player. Therefore, even if you are a scientifically and mathematically perfect violin player, it does not mean you are a great violin player.

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Results – Mersenne's Laws experiments – 1

Length/Frequency

Length of String ($m \times 10^{-2}$) ($\pm 1 \times 10^{-2} m$)	$\frac{1}{\text{Length of string}}$ ($m \times 10^{-2}$)	Error in Length of String ($m \times 10^{-2}$)	Frequency (Hz) ($\pm$ 10Hz)
8	0.125	$\pm$ 0.018	850
10	0.100	$\pm$ 0.011	706
12	0.083	$\pm$ 0.008	540
14	0.071	$\pm$ 0.006	486
16	0.063	$\pm$ 0.004	401
18	0.056	$\pm$ 0.003	389
20	0.050	$\pm$ 0.003	367

Tension/Frequency

Masses Applied (kg) ($\pm$ 0.001kg)	Tension (T) (N) ($\pm$ 0.0098N)	$\sqrt{T}$ (N)	Error in $\sqrt{T}$ (N)	Frequency (Hz) ($\pm$ 10Hz)
0.01	0.098	0.313	$\pm$ 0.016	132
0.02	0.196	0.443	$\pm$ 0.011	195
0.03	0.294	0.542	$\pm$ 0.009	294
0.04	0.392	0.626	$\pm$ 0.008	306
0.05	0.490	0.700	$\pm$ 0.007	337
0.06	0.588	0.767	$\pm$ 0.006	351
0.07	0.686	0.828	$\pm$ 0.006	375
0.08	0.784	0.885	$\pm$ 0.006	435

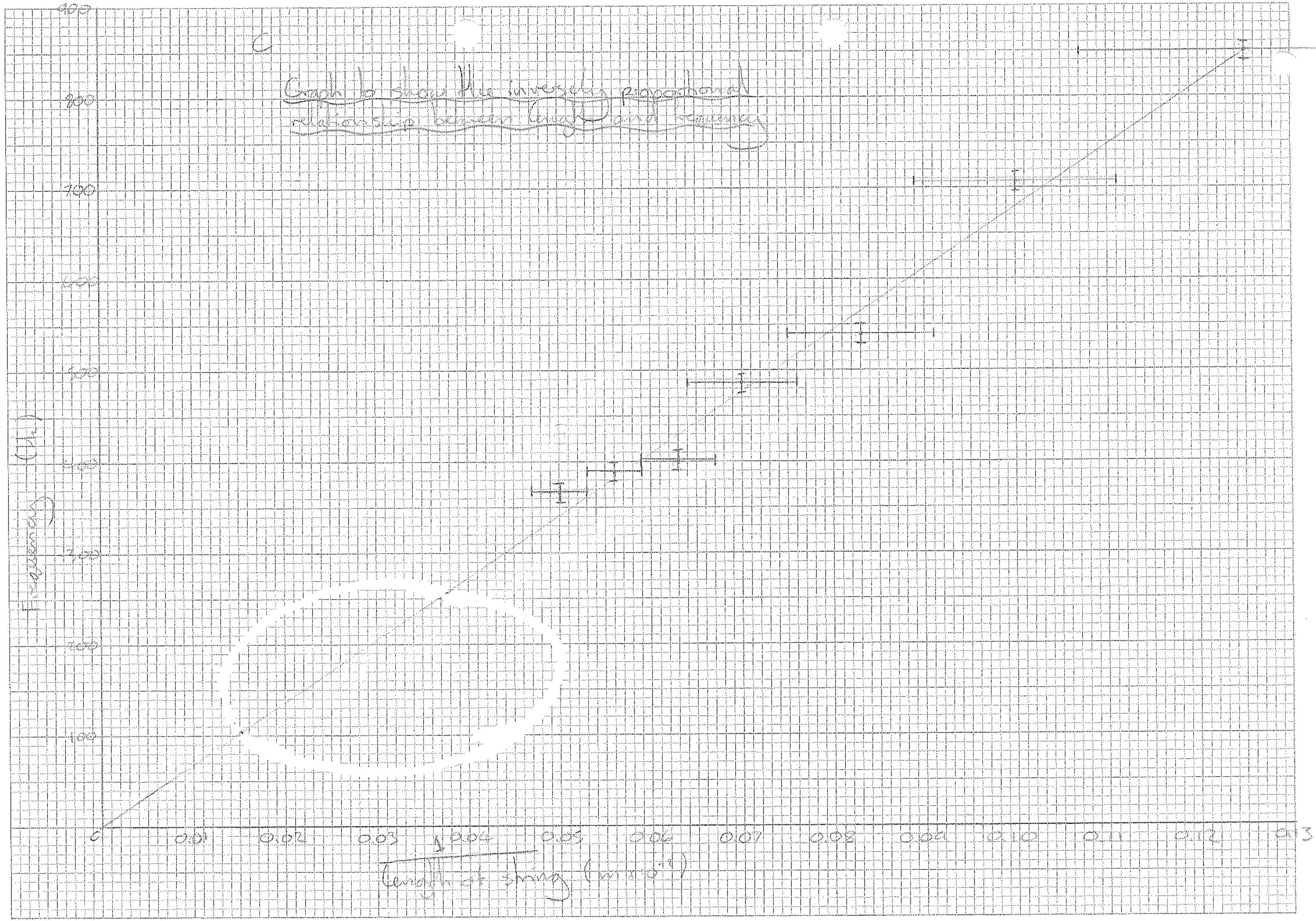
Results - Mersenne's Laws experiments - 2

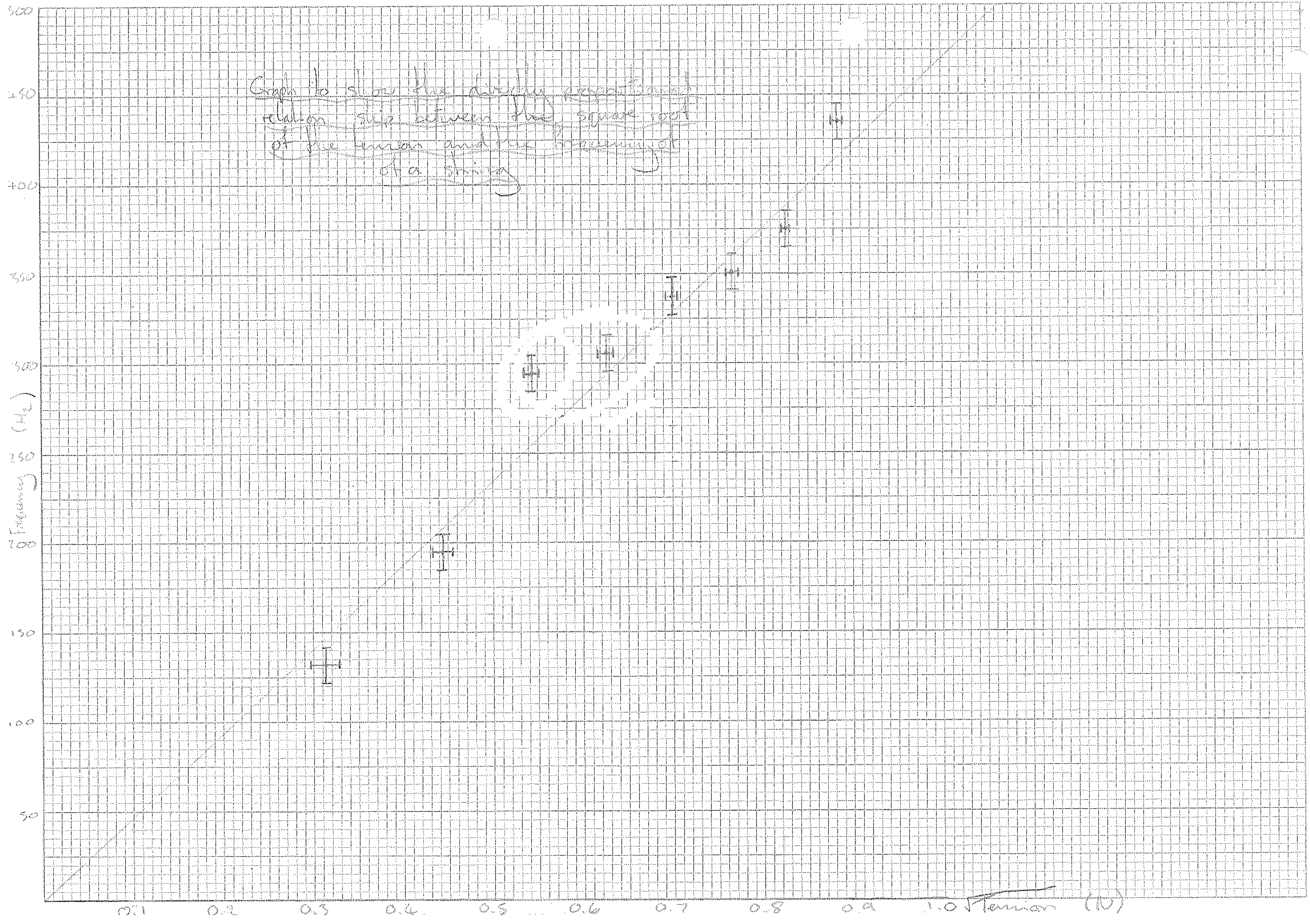
Diameter / Frequency

Frequency (Hz) (± 10 Hz)	Diameter (d) of string (m x 10^{-4}) (± 1 x 10^{-5} m)	$\frac{1}{d}$ (m)	Error in $\frac{1}{d}$ (m)
196	10.8	926	± 9
294	6.6	1515	± 23
440	4.4	2273	± 52
659	3.2	3125	± 101

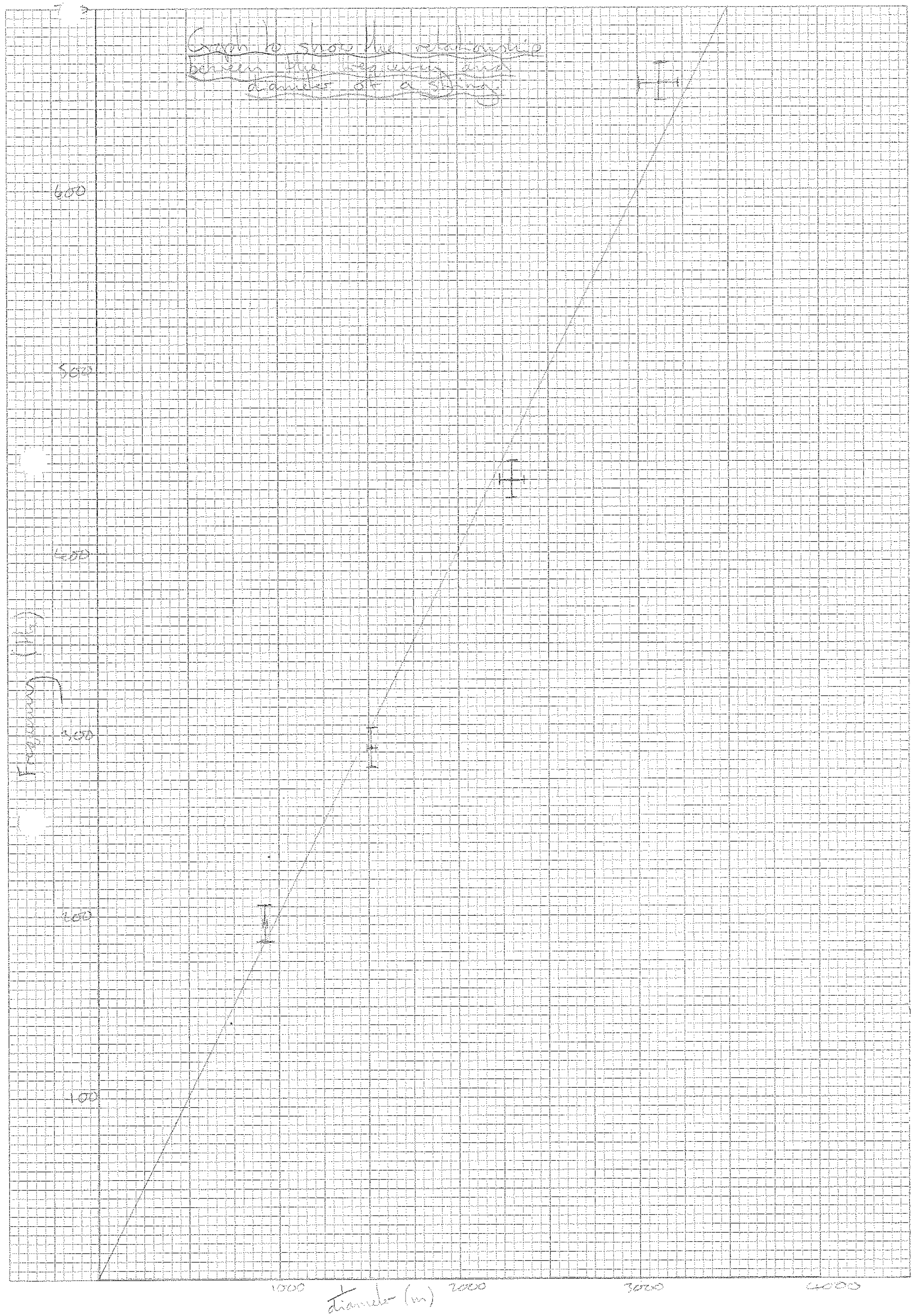
Mass per unit length / Frequency

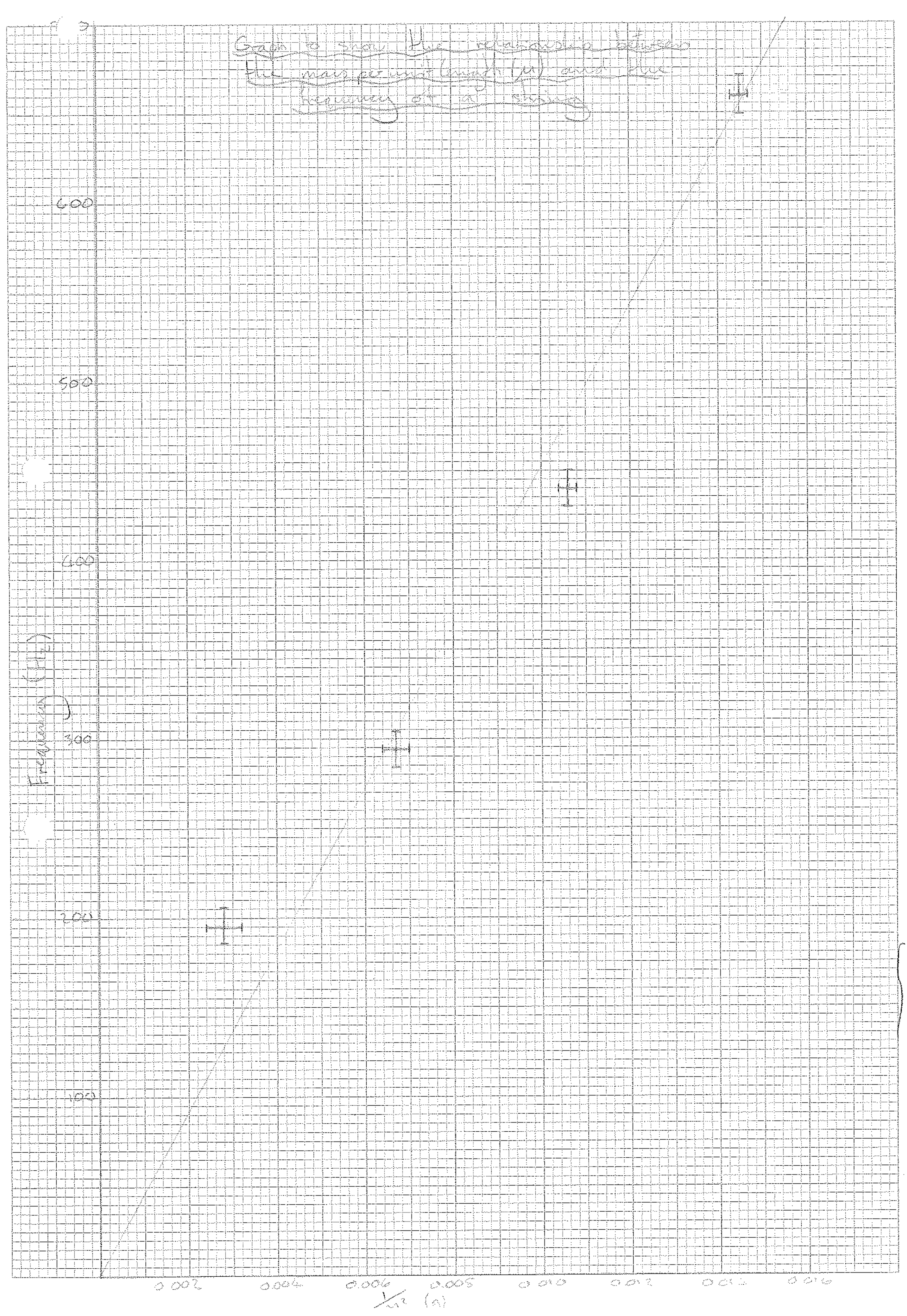
Frequency (Hz) (± 10 Hz)	Mass of 10 cm of string (g) (± 0.01 g)	Mass of 1 m of string (g) (± 0.1 g)	Mass of 1 m of string^2 (g) (μ^2)	Error in μ^2 (g)	$\frac{1}{\mu^2}$ (g)	Error in $\frac{1}{\mu^2}$ (g)
659	0.83	8.3	68.89	1.67	0.0145	0.0004
440	0.97	9.7	94.09	1.45	0.0106	0.0003
294	1.22	12.2	148.84	2.45	0.0067	0.0001
196	1.89	18.9	357.21	3.79	0.0028	0.0001


Graph to show the inversely proportional relationship between length and frequency
Frequency (Hz)
Length of string
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
0.11
0.12
0.13
100
200
300
400
500
600
700
800
900



Graph to show the directly proportional relationship between the square root of the tension and the frequency of a string
Frequency (Hz)
500
450
400
350
300
250
200
150
100
50
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
√Tension (N)


Graph to show the relationship between the frequency and diameter of a string
600
500
400
300
200
100
Frequency (Hz)
1000
2000
3000
4000
diameter (m)



Graph to show the relationship between
the mass per unit length (μ) and the
frequency of a string
600
500
400
300
200
100
Frequency (Hz)
0.002
0.004
0.006
0.008
0.010
0.012
0.014
0.016

Tuning experiment Calculation

Distance from bridge to nut :- 32.3×10^{-2} m

Mid-point between bridge and nut :- 16.15×10^{-2} m

Frequency of E-string (E_5) :- 659.26 Hz

Frequency of E_6 (Octave higher than E-string) :-

$$659.26\times2 = 1318.52\ \text{Hz}$$

or

$$\frac{32.3\times10^{-2}}{16.15\times10^{-2}}\cdot659.26 = 1318.52\ \text{Hz}$$

Minimum frequency difference my sister could detect :-

1 Hz.

Distance from bridge (or nut) when frequency is ±1 Hz :-

$$\frac{32.3\times10^{-2}\times659.26}{x} = 1319.52\ \text{Hz}$$

$$x = \frac{32.3\times10^{-2}\times659.26}{1319.52} = 16.13\times10^{-2}\ \text{m}$$

$\therefore$ The error with which a violinist can identify a wrong note is :-

$$16.15\times10^{-2} - 16.13\times10^{-2} = \pm\,0.012\times10^{-2}\ \text{m}$$

$$= \pm1.2\times10^{-4}\ \text{m}$$

Sound distribution around a violin

Results

Distance from left of violin (cm)	Volume of sound (arbitrary units)
10	22
20	18
30	12
40	10
50	8

Distance from right of violin (cm)	Volume of sound (arbitrary units)
10	20
20	17
30	15
40	10
50	7

Distance in direction of scroll (cm)	Volume of sound (arbitrary units)
10	13
20	8
30	3
40	1
50	1

Results (continued) - Sound Distribution around a violin

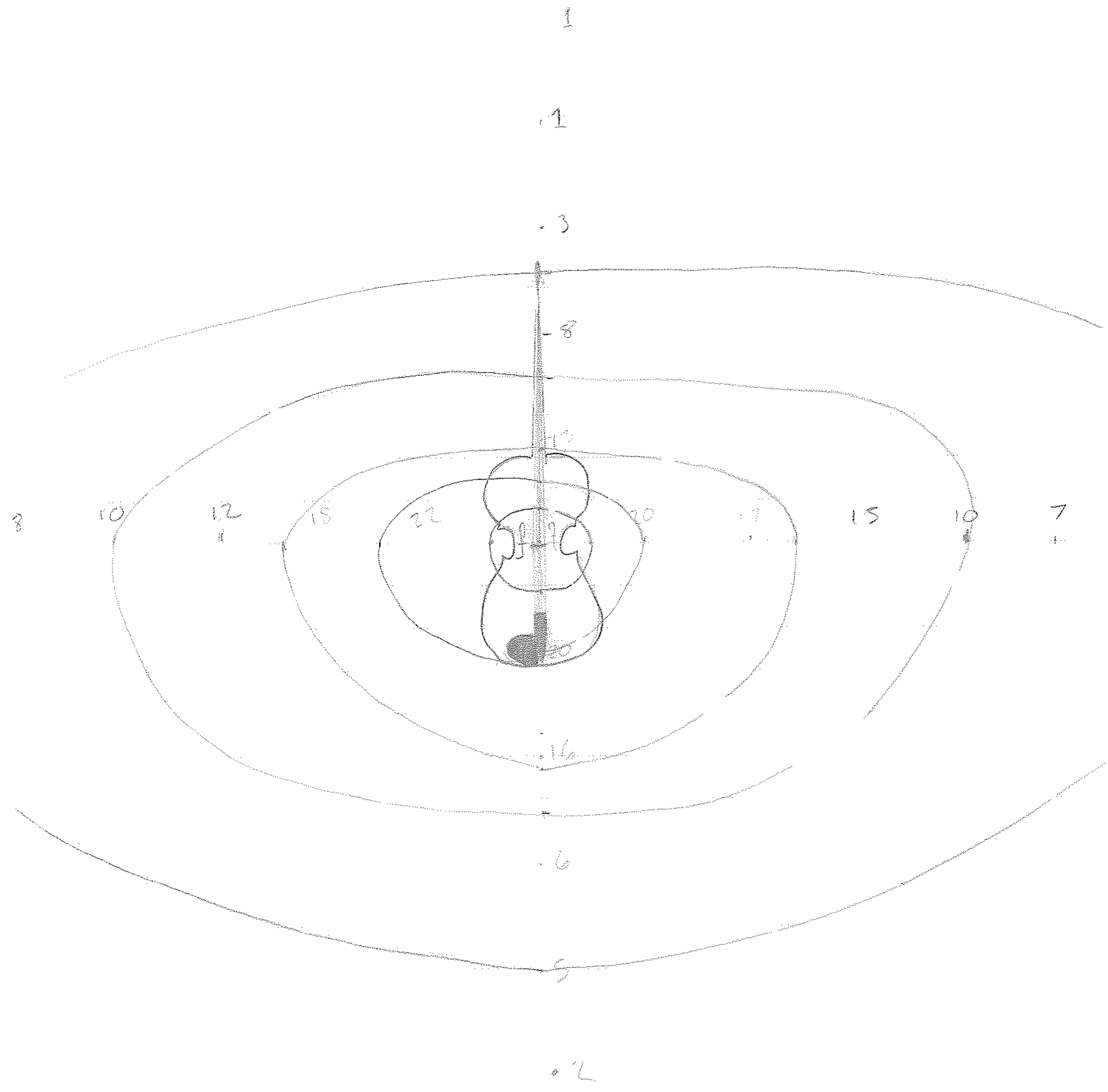
Distance in direction of chin rest (cm)	Volume of sound (arbitrary units)
10	20
20	16
30	6
40	5
50	2

Distance above violin (cm)	Volume of sound (arbitrary units)
10	26
20	20
30	15
40	11
50	9

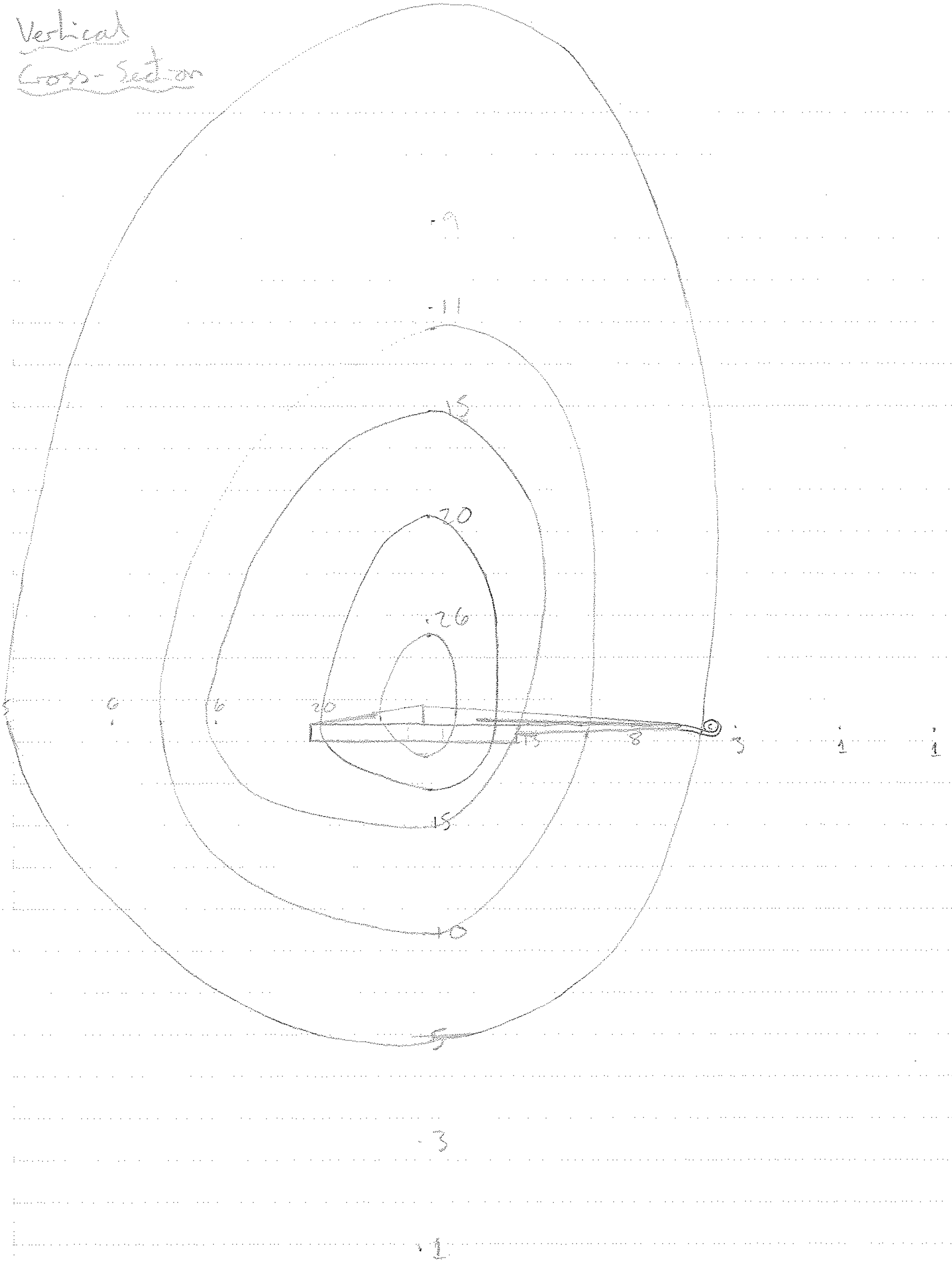
Distance below violin (cm)	Volume of sound (arbitrary units)
10	15
20	10
30	5
40	3
50	1

Horizontal

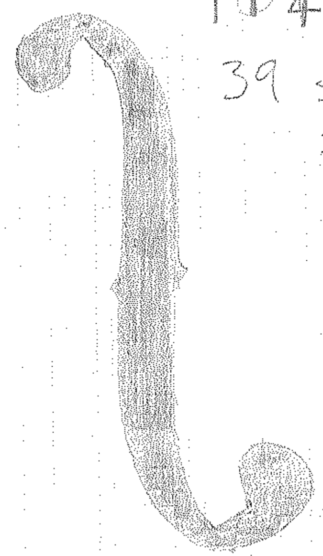
Cross-section



Vertical
Cross-Section



$\boxed{1 \text{ square } (\square) = 2\times10^{-3} \times 2\times10^{-3}}$
$= 4\times10^{-6}\ m^2$



104 complete squares
39 squares made up from partial squares

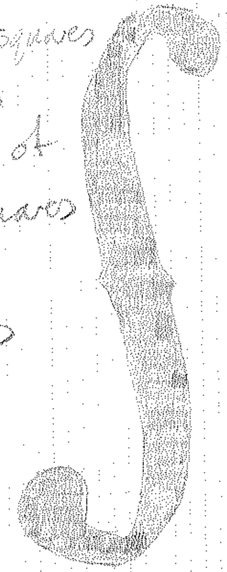
Total = 153 squares

Area = $153 \times 4\times10^{-6}$
$= 6.12\times10^{-4}\ m^2$

113 complete squares
38 squares made up of partial squares

Total = 151 squares

Area = $151 \times 4\times10^{-6}$
$= 6.04 \times 10^{-4}\ m^2$



Av. Area $= \dfrac{(6.12 + 5.6)\times10^{-4}}{2}$
$= 5.86\times10^{-4}\ m^2$

Resonant Frequency Equation + Calculation of speed of sound

$$f = \frac{0.27c}{V^{\frac{1}{2}}} A^{\frac{1}{4}}$$

$$y = m \quad x + c$$

Gradient

Results

frequency (Hz) (± 10Hz) (f)	Area of f-holes (m^2) ($\pm 1\times10^{-6} m^2$) (A)	$A^{\wedge \frac{1}{4}}$ (m^2)	Error In $A^{\wedge \frac{1}{4}}$ (m^2)
322.0	5.86×10^{-4}	0.1556	1×10^{-4}
312.5	5.27×10^{-4}	0.1515	1×10^{-4}
310.0	4.69×10^{-4}	0.1472	1×10^{-4}
307.5	4.10×10^{-4}	0.1423	1×10^{-4}
297.5	3.52×10^{-4}	0.1370	1×10^{-4}
277.5	2.93×10^{-4}	0.1309	2×10^{-4}
275.0	2.34×10^{-4}	0.1237	2×10^{-4}
260.0	1.76×10^{-4}	0.1152	2×10^{-4}
232.5	1.17×10^{-4}	0.1040	2×10^{-4}
180.0	5.90×10^{-5}	0.0876	4×10^{-4}

Centroid :- $\Sigma_{(\text{frequency})} = 1.295 \div 10 = 0.1295$

$\Sigma_{(A^{\wedge\frac{1}{4}})} = 2774.5 \div 10 = 277.45$

- (0.1295, 277.45)

From Graph :- $\text{Grad}_{(\text{max})} = 2138.2$, $\text{Grad}_{(\text{min})} = 1748.2$

$$\text{Grad}_{(\text{average})} = \frac{2138.2 + 1748.2}{2} = 1943.2$$

$$\text{Gradient} = \frac{0.27c}{V^{\frac{1}{2}}} = \frac{0.27c}{0.00185^{\frac{1}{2}}} = 6.27737254c$$

$$c = \frac{1943.2}{6.28} = 309.4 \text{ ms}^{-1}$$

Calculation of Speed of Sound (continued)

Maximum :-

$$c = \frac{2138.2}{6.28} = 340.6\,ms^{-1}$$

Minimum :-

$$c = \frac{1748.2}{6.28} = 278.4\,ms^{-1}$$

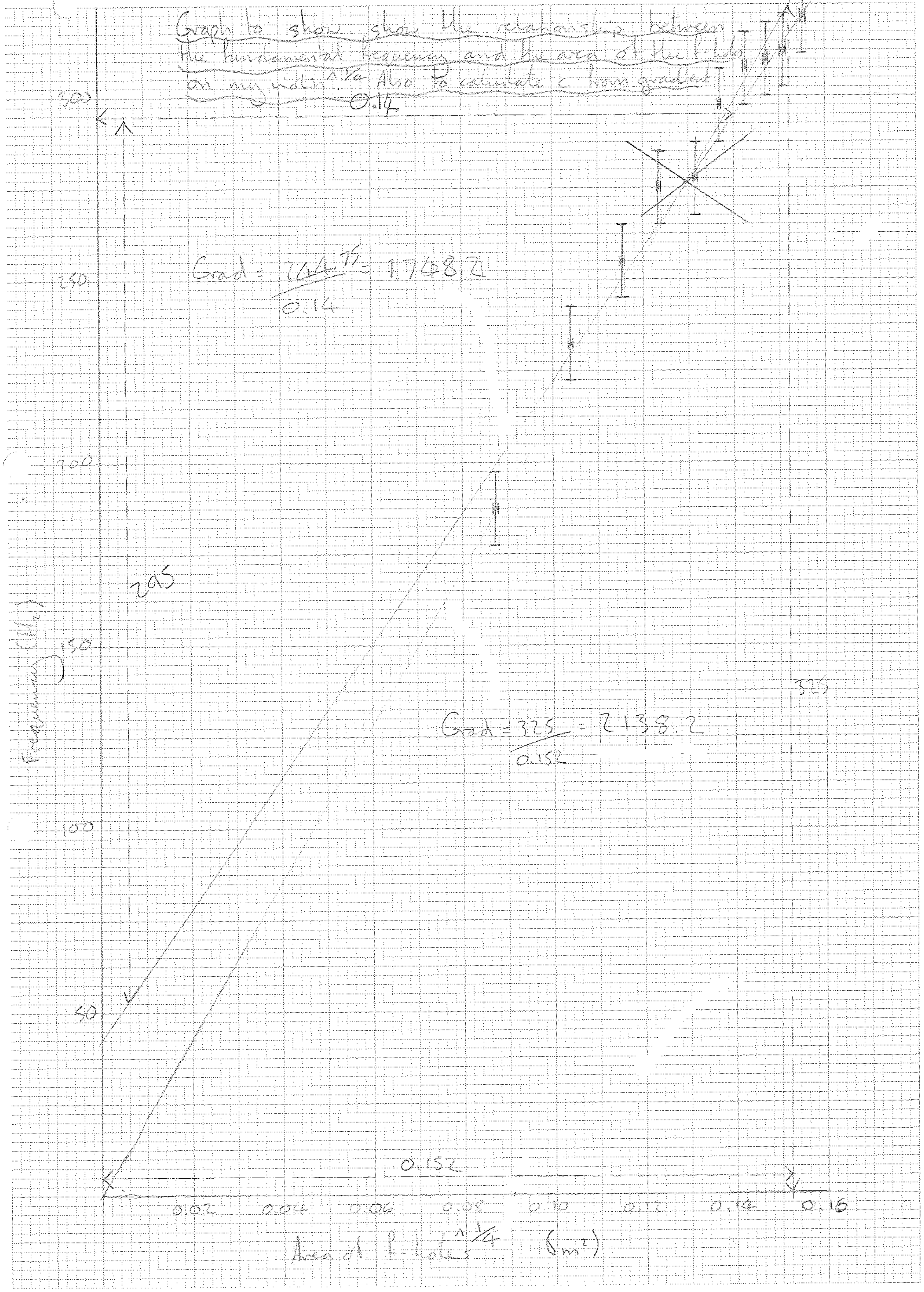
Error in c :-

$340.6 - 309.4 = 31.2$
$309.4 - 278.4 = 31.0$

$c = (309.4 \pm 31.2)\,ms^{-1}$

$$\% \text{ error} = \frac{31.2}{309.4} \times 100 = 10.1\%$$

$c = (309.4 \pm 10\%)\,ms^{-1}$


Graph to show show the relationship between the fundamental frequency and the area of the P-hole on my violin ^1/4 Also to calculate c from gradient
0.14
Grad = 244 / 0.14 = 1748.2
295
Grad = 325 / 0.152 = 2138.2
325
0.152
300
250
200
150
100
50
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
Area of F-holes ^1/4 (m²)
Frequency (Hz)